

Highlights

Title: "Second- and higher-order data generation and calibration: A tutorial"

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► Second- and third-order (multi-way) data and algorithms are reviewed. ► Suitable examples of different complexity are provided. ► The advantages of multi-way calibration are illustrated. ► Multi-way analytical figures of merit are discussed.

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3 **Second- and higher-order data generation and**

4 **calibration: A tutorial**

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24 **Abstract**

25 An introduction to multi-way calibration based on second- and higher-order data
26 generation and processing is provided, with emphasis on practical experimental aspects.
27 After a discussion concerning a proper nomenclature scheme, a suitable classification of
28 the obtainable data, and the general features of the available algorithms and their
29 underlying models, a series of examples is discussed in detail, with the purpose of
30 illustrating the great potentiality of the field for the analytical community. Emphasis is
31 directed towards the most popular multi-way data, i.e., second-order or matrix data,
32 which can be conveniently measured in a variety of instruments. Third-order data are
33 being increasingly studied and are also discussed, along with the less explored field of
34 fourth-order data. The estimation of figures of merit, which analysts need to report
35 during method development, is now sufficiently mature to be provided for the general
36 audience.

37

38 *Keywords:* Multi-way calibration; Parallel factor analysis; Multivariate curve
39 resolution; Partial least-squares

40

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66 **1. Introduction**

67 The variety of second- and higher-order instrumental data which are being produced
68 by modern instruments, and the subsequent enhancement in analytical properties which
69 is obtained by processing this kind of data, have made multi-way analysis a subject of
70 high interest for the analytical community. Multi-way analysis has produced a
71 significant impact on the development of analytical methods, especially for the
72 quantitation of analytes of interest in complex matrices, such as those found in
73 environmental, biological and food samples, among others [1-3]. The subject started
74 with the pioneering work of Kowalski [4], among others, and specifically with the first
75 experimental realization of a multi-way calibration of complex samples in the presence
76 of interferences in 1978 [5]. A literature search reveals that more than 360 papers have
77 been published in the last five years on the subject, including interesting comprehensive
78 reviews [6-9].

79 The emphasis directed to processing complex data has been accompanied by the
80 development of a diversity of mathematical algorithms based on various data models,
81 which are available to analytical chemists for the convenient study of this body of
82 information. A casual reader may be confused by the variety of experimental and
83 theoretical developments which have been flourishing in this area in recent years. This
84 provides the main motivation for the present tutorial, i.e., to consistently classify and
85 compare the various participants of the scene: different types of second- and higher-
86 order instrumental data and the various available models and algorithms, as well as the
87 selection of the most convenient ones for a given experimental application. In this
88 context, several examples extracted from the literature are discussed, showing details
89 regarding the data generation and the advantages and disadvantages of the application of

90 some specific models and algorithms. In addition, a discussion about recent
91 developments concerning analytical figures of merit is included.

92 An important message is to be left from the present tutorial to the analytical
93 community, which is of paramount importance and is perhaps the major legacy of
94 multi-way calibration. It may be better understood using an example: suppose one wish
95 to quantitate an analyte in a sample from absorbance measurements at a single
96 wavelength, but there are other constituents whose responses overlap with that of the
97 analyte. It is well-known that this is not possible in this univariate context. If spectra
98 were measured at many wavelengths (i.e., vectorial absorbance data), it would be
99 possible to accomplish the goal, but only on the condition that a calibration model is
100 produced from a large set of samples, which contain all varying concentrations of
101 possible future sample constituents. However, if matrix data (or higher-order arrays) are
102 recorded, and sufficient selectivity is present in the various data modes, it is in principle
103 possible to predict analyte concentrations in any future sample, no matter how many
104 signal-overlapping constituents this sample contains, and having calibrated with *pure*
105 *analyte standards*. This is the experimental realization of the main advantage offered by
106 second- and third-order calibration, i.e., the so-called 'second-order advantage'.

107 The tutorial is organized as follows: an introduction to multi-way nomenclature is
108 first provided, along with a discussion on data properties, models and algorithms,
109 followed by a series of examples concerning the most explored data for multi-way
110 calibration, such as luminescence matrix spectroscopy and chromatography with
111 multivariate detection. A final section is devoted to the latest developments in the
112 estimation of multi-way analytical figures of merit.

113

114 **2. Nomenclature**

115 In this section the usual nomenclature concerning sample constituents, algorithms,
116 methods and data is presented. It should help the general reader to follow the remaining
117 of the tutorial.

118 It is first important to classify the different constituents which may be present in the
119 various sample types which are normally studied during multi-way calibration.
120 Constituents present in the samples employed for calibration and validation are
121 regularly called 'expected', i.e. they are included in these sets because they are expected
122 to be present in future samples. On the other hand, constituents which are only present
123 in the unknown samples are called 'unexpected', and also 'potential interferents'. The
124 expected constituents can be further sub-classified into 'calibrated' and 'uncalibrated':
125 the former are those for which calibration concentrations are known, whereas the latter
126 are constituents producing measurable signals but whose concentrations are not known
127 [1,10].

128 The so-called potential interferents, however, will not always generate an
129 interference, leading to a systematic error in the analyte quantitation [11]. Depending on
130 the measured instrumental signals and calibration methodology, the interference may
131 only remain as potential. In first-order calibration, for example, unexpected constituents
132 are usually true interferences. However, in higher-order calibration achieving the
133 second-order advantage, the signal from the unexpected constituents can be modelled
134 and mathematically removed, in such a way that their effect is negligible [12].

135 As a general note, we prefer to refer to chemical sample *constituents* rather than to
136 *components*, because the latter word may imply abstract linear combinations of real
137 constituents for some methodologies.

138 It is also important to distinguish among *technique*, *method*, *model*, and *algorithm*,
139 terms which are sometimes interchanged. In the context of calibration, an analytical
140 technique is a procedure used to determine an analyte concentration, whereas an
141 analytical method is more specific concerning the sample and measuring conditions. A
142 model is a description of the data properties, and an algorithm is a detailed set of
143 instructions for accomplishing a computational task. Therefore, the specific
144 mathematical procedures for processing second- and higher-order data are all
145 algorithms. They allow for data analysis based on a certain model, i.e., on certain
146 assumptions concerning the properties of the data. Different algorithms may apply to
147 the same model. The term *method* is employed in a more general sense and sometimes
148 replaces the term *algorithm* [1].

149 Table 1 shows the natural progression from the simplest zeroth-order to multivariate
150 data. Zeroth-order corresponds to instruments producing a single response per sample (a
151 zeroth-order tensor), such as the absorbance at a single wavelength or the reading of an
152 ion-selective electrode. First-order data for a given sample are arranged as a vector or
153 first-order tensor, such as spectra: UV-visible spectrophotometry, spectrofluorimetry,
154 infrared, near infrared (NIR), nuclear magnetic resonance, electrochemical scans
155 (voltammograms, chrono-amperograms), among others. On the other hand, matrix data
156 for a single sample are considered to be second-order. They can be recorded in two
157 ways: (1) using a single instrument, such as a spectrofluorimeter registering excitation-
158 emission matrices (EEMs) or a diode-array spectrophotometer following the kinetics of
159 a chemical reaction, or (2) coupling two 'hyphenated' first-order instruments, as in
160 tandem gas chromatography-mass spectrometry (GC-MS), GC-GC, MS-MS, etc. When
161 second-order data for a group of samples are joined into a single, three-dimensional
162 array, the resulting object is known as three-way array and these data are usually known

163 as three-way data. Finally, introducing an extra mode in the data leads to higher-order
164 data, in which case the mathematical object obtained by grouping third-order data for
165 several samples into the fourth direction, for example, is known as a four-way array.
166 Examples of four-way arrays are those obtained by following the kinetics of an
167 excitation-emission (EEM) fluorescence data matrix, or by hyphenating three
168 instruments, e.g. two-dimensional liquid chromatography with diode array detection
169 (LC-LC-DAD).

170 Notice that two alternative nomenclatures are employed for describing data and the
171 associated calibrations. One could refer to either second-order calibration or three-way
172 calibration; the former expression focuses on the number of modes of a single sample
173 (two modes, second-order data) whereas the latter on the number of modes of a sample
174 set (three modes, three-way data). The analytical community is used to the order-based
175 nomenclature, probably because of the expression 'second-order advantage'. However,
176 the chemometric community prefers the way-based nomenclature. In any case, it should
177 be clear from the context which is the number of working modes and the type of multi-
178 way calibration being discussed in each of the tutorial sections.

179

180 **3. Models and algorithms**

181 There are many algorithms available for processing multi-way data, but a few
182 underlying models on which they are based. The choice of a particular model and
183 algorithm should be primarily guided by the properties of the data, in the sense that the
184 model should match the data properties. Below we summarize the most popular multi-
185 way algorithms and their underlying models, with emphasis on which type of data can
186 be analyzed by them.

187

188 3.1. Second-order data

189 An important property of second-order data is the trilinearity. A group of measured
190 data matrices for a set of samples can in principle be arranged into a three-way data
191 array, as shown in Fig. 1A. The array is considered to be trilinear if its elements can be
192 reasonably fit to the following expression:

$$193 \quad x_{ijk} = \sum_{n=1}^N a_{in} b_{jn} c_{kn} + e_{ijk} \quad (1)$$

194 where a_{in} represents the relative concentration (also called *score*) of a given constituent
195 n in the i -th sample, b_{jn} and c_{kn} are the intensities in both of the instrumental modes j
196 and k , respectively (also called *loadings*) and e_{ijk} collects the fitting errors. Summation
197 in equation (1) implies that the individual constituent signals are additive. Usually the
198 loadings are normalized to unit length, and collected into the loading matrices **B** and **C**,
199 of size $J \times N$ and $K \times N$ respectively (Fig. 2), and the scores are collected into the score
200 matrix **A** (size $I \times N$). Strictly speaking, an array is trilinear when the number of trilinear
201 components (N) required to describe the data through equation (1) is small, and is
202 ideally equal to the number of chemical constituents producing measurable signals. In
203 the remainder of this work we will simply refer to such arrays as trilinear. In principle, a
204 non-trilinear array can also be described using equation (1), but will require a value of N
205 significantly larger than the actual number of responsive constituents.

206 Equation (1) represents the trilinear model which is the basis of trilinear
207 decomposition algorithms. The fitting of a trilinear three-way array to this model often
208 provides unique solutions, meaning that there is a single set of **A**, **B** and **C** matrices
209 whose elements satisfy equation (1) to a reasonable degree. The uniqueness of the
210 decomposition is the natural basis of the second-order advantage, because the profiles

211 contained in matrices **B** and **C** are proportional to the true instrumental profiles of each
212 pure sample constituent in each data mode (except for a scaling factor), and the scores
213 contained in **A** are proportional to the pure constituent concentrations, as if the
214 constituents were physically separated from the sample. Usually the data for the
215 calibration samples and each test sample are joined into a single three-way array. The
216 information provided by the constituent scores in the calibration samples can be used in
217 the context of calibration to build a plot of analyte scores vs. nominal analyte
218 concentrations. The analyte concentration in the unknown sample is then predicted by
219 interpolating its score in the fitted line. Similar calibration plots are also employed in
220 the context of first-order multivariate calibration [13,14].

221 **** Insert Fig. 1 ****

222 Algorithms based on the trilinear model are thus useful for multi-way calibration
223 from trilinear three-way data. One of the most employed trilinear algorithms is parallel
224 factor analysis (PARAFAC) [15-18]. Although there are various versions of PARAFAC
225 for fitting equation (1) [15,19], we herein refer to the one which uses an alternating
226 least-squares (ALS) procedure. PARAFAC has become the algorithm of choice for
227 calibration based on trilinear three-way data analysis, due to its efficiency, robustness,
228 ability to process multiple samples, and availability of a variety of constraints to be
229 applied during the fit, which ensures reaching physically interpretable results.
230 Additional algorithms based on trilinear modelling of three-way arrays are self-
231 weighted alternating trilinear decomposition (SWATLD) [20] and penalized alternating
232 trilinear decomposition (APTLD) [21]. Other less employed algorithms in this context
233 are generalized rank annihilation (GRAM) [22], direct trilinear decomposition (DTLD)
234 [23] and bilinear least-squares (BLLS) [24], either because the use single calibration
235 standards (GRAM, DTLD) or do not achieve the second-order advantage (BLLS).

236 One common cause by which a three-way array deviates from trilinearity is the
 237 presence of profiles of the constituents in one of the data modes which change from
 238 sample to sample. This is typical of chromatographic-spectral data, because the spectra
 239 can be reasonably expected to be constant for a given constituent in different samples,
 240 but the elution time profiles are likely to change due to uncontrolled irreproducibility in
 241 chromatographic runs, particularly in liquid chromatography. This may also happen for
 242 other instrumental modes: pH gradients may show differences from run to run, kinetic
 243 profiles may differ because of uncontrolled temperature changes, and even spectra may
 244 change if interactions occur with the sample background. The mode with low
 245 reproducibility across samples is said to be the trilinearity-breaking mode, because if the
 246 profiles in this mode for all constituents were equal in all samples, the data would be
 247 trilinear [25]. In the latter case PARAFAC could be conveniently applied; however if
 248 lack of reproducibility indeed occurs, calibration using trilinear models would not be
 249 recommended.

250 ** Insert Fig. 2 **

251 A three-way array of non-trilinear data of this type can be unfolded into an
 252 augmented matrix, as shown in Fig. 1B. If the augmentation mode is chosen to be the
 253 elution time mode (i.e., the trilinearity breaking mode in this particular case), then the
 254 resulting augmented matrix \mathbf{X}_{aug} is of size $IJ \times K$, if J is the number of data points in the
 255 elution time mode for each of the I data matrices involved in \mathbf{X}_{aug} . This latter matrix is
 256 bilinear, and thus its elements can be fitted to a bilinear model:

$$257 \quad x_{\text{aug},mk} = \sum_{n=1}^N b_{\text{aug},mn} c_{kn} + e_{mk} \quad (2)$$

258 where $x_{\text{aug},mk}$ is a generic element of the augmented matrix \mathbf{X}_{aug} (the index m runs from
 259 1 to IJ), $b_{\text{aug},mn}$ is an element of the augmented profile for constituent n in the augmented
 260 direction, c_{kn} an element of the profile in the direction for constituent n (this profile is

261 common to all samples), and e_{mk} is an element of the matrix of model residuals. The
262 elements $b_{aug,mn}$ and c_{kn} are usually collected into a matrix \mathbf{B}_{aug} of augmented profiles
263 and a matrix \mathbf{C} of normalized profiles.

264 In contrast to the trilinear decomposition of a trilinear three-way array into the three
265 matrices \mathbf{A} , \mathbf{B} and \mathbf{C} (see above), the bilinear decomposition of a bilinear augmented
266 matrix into \mathbf{B}_{aug} and \mathbf{C} is not unique. However, equation (2) can often be solved in terms
267 of true constituent profiles if a proper set of initial values for \mathbf{B}_{aug} or \mathbf{C} are found in
268 order to start the bilinear decomposition, and suitable constraints are applied during the
269 fit, to limit the number of possible solutions, and to ensure they are chemically
270 reasonable. A popular algorithm accomplishing this goal is multivariate curve resolution
271 coupled to ALS (MCR-ALS) [26-28]. MCR-ALS is usually employed for quantitative
272 analytical purposes in the so-called extended mode [29], which decomposes an
273 augmented data matrix created from calibration and unknown matrices. Many different
274 constraints are available in MCR-ALS, while initial values can be efficiently estimated
275 by a variety of methods [30,31].

276 Once a reasonable solution has been found by MCR-ALS by proper initialization and
277 natural constraints, pure constituent concentration information is contained in the areas
278 under each of the \mathbf{B}_{aug} profiles for each sub-matrix corresponding to each of the
279 participating samples (Fig. 3). This allows one to achieve the second-order advantage,
280 because information on potential interferents is efficiently separated from those for the
281 analytes. A univariate plot can be built by regressing the areas under the profiles for a
282 specific analyte vs. its nominal concentration in the calibration samples. The analyte can
283 then be predicted in unknown samples by interpolating, in the univariate plot, the area
284 of the \mathbf{B}_{aug} profile corresponding to that particular analyte in the unknown sample.

285 ** Insert Fig. 3 **

311

312 3.2. Third-order data and beyond

313 The models followed by third-order data are derived by analogy with second-order
314 data, namely: (1) quadri-linear, which is the logical sequel after trilinear, (2) a bilinear
315 augmented matrix for non-quadrilinear data with quadri-linearity breaking modes, and
316 (3) latent variable models for other, more complex, non-quadrilinear data.

317 With third-order data for a group of samples, a four-way data array can be
318 constructed, in which case the simplest model is the quadri-linear one, or multi-linear in
319 general. Multi-linearity can be defined by extension of equation (1), and a four-way data
320 array is quadri-linear if its elements comply with:

$$321 \quad x_{ijkl} = \sum_{n=1}^N a_{in} b_{jn} c_{kn} d_{ln} + e_{ijkl} \quad (3)$$

322 where all symbols are as in equation (1), with d_{ln} describing the profile in the fourth data
323 mode. As for the decomposition of a trilinear three-way array, uniqueness is often
324 achieved in the decomposition of a quadri-linear four-way array.

325 If the data are quadri-linear then multi-way PARAFAC would be adequate, because
326 it can be applied to data with any number of ways, employs multiple calibration
327 samples, and includes useful constraints during the fitting phase. Complementary
328 algorithms such as alternating penalty quadri-linear decomposition (APQLD) [41] and
329 alternating weighted residual constraint quadri-linear decomposition (AWRCQLD) [42]
330 are also available for processing quadri-linear data. Excitation-emission luminescence
331 matrices measured as a function of reaction or decay time are prime examples of quadri-
332 linear data to which these algorithms can be confidently applied (see below).

333 As with second-order data, a common cause of multi-linearity loss is the variation of
334 constituent profiles in one particular mode from sample to sample, a phenomenon

335 usually observed in multi-way data of chromatographic origin. To be able to apply
336 MCR-ALS to these data, it is necessary to first unfold the original third-order data into
337 matrices, so that they could then be arranged into a bilinear augmented matrix. A typical
338 case is two-dimensional chromatography with spectral detection, in which the data
339 modes are two elution times and spectra. Since the former two are potentially quadri-
340 linearity breaking, it is necessary to unfold the two chromatographic elution time modes
341 into a single one, transforming the original three-way arrays into matrices (Fig. 5). This
342 approach has been taken when MCR-ALS is applied to third-order chromatographic
343 data of this kind [43,44].

344 **** Insert Fig. 5 ****

345 More complex data deviating from quadri-linearity by other causes can be analyzed
346 using latent structures, coupled to residual trilinearization (RTL) to achieve the second-
347 order advantage, as the natural extension of RBL to three data modes. This gives rise to
348 unfolded- and multi-way PLS combined with RTL (U-PLS/RTL, N-PLS/RTL) [45,46].
349 The hybrid technique trilinear least-squares/RTL (TLLS/RTL) is also available but less
350 flexible than the former ones [46]. For one way further, the combination of U-PLS with
351 residual quadri-linearization (U-PLS/RQL) has been recently reported, and is available
352 for processing five-way data arrays [47].

353 **3.3. Software**

354 Software for multi-way analysis is freely available on the Internet, in the form of
355 MATLAB codes [48], including several useful graphical user interfaces (GUI) [49-52].
356 Table 2 shows a variety of free MATLAB programs for multi-way data processing.

357

358 **3.4. Summary**

359 To summarize the above discussion on models and algorithms, multi-way data
360 should be first classified by the analyst according to their properties. There are three
361 main data types in this regard: (1) multi-linear data, (2) non multi-linear data but
362 unfoldable into a bilinear data matrix, and (3) other non multi-linear data which cannot
363 be unfolded into a bilinear data matrix. This classification gives rise to the three main
364 data models on which data processing tools should resort: (1) the multi-linear model, (2)
365 the bilinear augmented matrix, and (3) latent variable models.

366 Once this classification is made, the subsequent task is to find a suitable data
367 processing algorithm. The recommendation in this regard is to resort to an algorithm
368 fulfilling the following requirements: (1) be based on the model the data are assumed to
369 follow, and (2) allow one to perform calibration using multiple standard samples. For
370 the three data types mentioned above, then, three algorithm types can be recommended:
371 (1) PARAFAC and its TLD/QLD variants for multi-linear data, (2) MCR-ALS for non-
372 multilinear data which are unfoldable into a bilinear augmented matrix, and (3)
373 PLS/RML for the remaining non-multilinear data. All these algorithms allow one to
374 calibrate using as many standards as desired, and are thus preferable over single-
375 calibration-sample algorithms of any kind.

376

377 **4. Second-order data generation and examples**

378 Luminescence (fluorescence, phosphorescence, chemiluminescence) and other
379 spectroscopic data, such as UV-visible spectrophotometric data, are prone to be affected
380 by spectral interferences and, therefore, the coupling with multivariate calibration is an
381 excellent alternative to gain selectivity in this area by mathematical means.

382 To obtain second-order data from UV-visible spectrophotometric measurements,
383 they are usually combined with kinetic or pH-gradient experiments: the absorbance
384 spectra constitute one data mode and the reaction time (or pH) the other one. Some
385 application examples can be found in the literature regarding these data types [53,54]. A
386 more general way is by multi-wavelength (diode array) detection in chromatography,
387 which is discussed in a separate section.

388 Among luminescence signals, fluorescence is by far the most frequently employed,
389 although examples on the use of time-resolved phosphorescence at room temperature
390 have been also reported [55]. Due to the important advantages of methods based on
391 fluorescence emission, they are profusely used with analytical purposes for the
392 determination of many analytes which display either native or induced fluorescence
393 emission. Among these advantages, the following can be mentioned: high sensitivity
394 (which is intrinsic to the fluorescence signals) and low or null consumption of organic
395 solvents (allowing one to work under green-chemistry principles). On the other hand, if
396 selectivity is an issue, it can be improved by coupling to second-order calibration.

397 Examples of luminescence second-order data are excitation-emission fluorescence
398 matrices (EEFM) [56], reaction time (kinetic)-excitation or emission wavelength
399 matrices [57], and chromatography with spectral luminescence detection. In what
400 follows, we discuss literature examples where second-order calibration is applied to
401 luminescence data, with emphasis on different data sets and the specific algorithms that
402 can be applied in each case. The overall idea of the following sections is to discuss
403 examples showing a variety of experimental possibilities which can be found in
404 practice. Although the general rule (see above) is that luminescence data are trilinear
405 and hence PARAFAC (or its TLD variants) is the algorithm of choice for processing
406 them, there are some instances in which this is not the case. Hence the discussed

407 examples should warn analytical chemists to be cautious in what concerns algorithm
408 selection.

409

410 **4.1. Trilinear luminescence data**

411 As discussed before, an individual EEFM is an example of bilinear data, and in many
412 cases, a set of EEFMs can be arranged into a trilinear three-way array, in which case
413 trilinear decomposition methods can be applied to retrieve the excitation and the
414 emission spectra for the participating fluorescent constituents. However, RBL and
415 MCR-ALS can also be conveniently employed to process these data, because they are
416 more general and include trilinearity as a specific case.

417 The most popular trilinear algorithm is probably PARAFAC (see above), which in
418 addition to being friendly for analysts, has the advantage of providing important
419 physical interpretation, in the form of the fluorescence profiles of the constituents under
420 study. Many literature examples using PARAFAC to resolve systems based on EEFMs
421 can be cited [56]. Only as a recent example, we mention the spectrofluorimetric
422 determination, in environmental water samples, of the herbicide bentazone (BTZ), one
423 of the most employed herbicides in countries of profuse agriculture [58].

424 The EEFMs were measured in the presence of methyl- β -cyclodextrin, which
425 produces a significant increasing of the bentazone fluorescence intensity. The
426 experiment involved the following steps, which are the usual ones in the development
427 and validation of multi-way analytical procedures: (1) measurement of EEFMs of
428 standard solutions of BTZ of known concentrations (calibration step), (2) measurement
429 of EEFMs of solutions of BTZ different from those for calibration, and verification of
430 the predictive ability in the absence of interferents (validation step), (3) measurement of
431 EEFMs of artificial samples prepared by adding to BTZ solutions different amounts of

432 seven common agrochemicals (fuberidazole, thiabendazole, dichlorophene,
433 carbendazim, carbaryl, carbofuran and 1-naphthaleneacetic acid) acting as potential
434 spectral interferences (these samples are used for testing the predictive ability in the
435 presence of unexpected constituents), and (4) measurement of EEFMs of real samples
436 containing the investigated analyte. All the studied samples could be satisfactorily
437 resolved by PARAFAC, because the systems retained their trilinearity even in the
438 presence of interfering agents in both synthetic and real samples.

439 The wavelength ranges for calibration, validation and real samples (390-470 nm and
440 240-372 nm for emission and excitation, respectively) were selected in order to cover
441 the regions of fluorescence emission and excitation of BTZ. On the other hand, because
442 the constituents selected as interferents display intense fluorescence signals and were
443 added at high concentrations in order to check for potentially unfavorable situations, a
444 saturation of the fluorescence signal was observed in a wide spectral region of the
445 EEFMs for the test samples. Therefore, a restricted region, where the instrumental
446 signal could be correctly measured, was selected for data processing (390-470 nm and
447 340-372 nm for emission and excitation, respectively).

448 The number of responsive components was selected applying three procedures: (1)
449 taking into account the results of the core consistency analysis, (2) through the analysis
450 of PARAFAC residuals, and (3) considering that the addition of subsequent components
451 did not generate repeated profiles (we refer here to 'components' because the algorithm
452 is blind to whether they are real chemical constituents or not). It is important to note that
453 the core consistency analysis is a tool based on some data structural assumptions and
454 may fail in certain circumstances [59]. In other words, in addition to the core
455 consistency analysis, other tools to provide evidence of the appropriate number of
456 components should be considered [56].

457 The results obtained by the three procedures for determining the number of
458 components required by PARAFAC were consistent and established that this number in
459 the validation and test samples was two. In the validation samples, the number was
460 ascribed to the analyte and to a background signal. In the test samples, the number was
461 assigned to BTZ and to a combined signal corresponding to the interferents (taking into
462 account that in this restricted wavelength zone the background signal is negligible and
463 does not significantly contribute to the total signal). In this latter case, PARAFAC was
464 not able to discern between the profiles of each foreign constituent, and retrieved the
465 interference profiles as a single unexpected constituent. However, this fact did not
466 preclude the obtainment of good analytical results in these samples. In spiked natural
467 water samples, the number of PARAFAC factors was two or three, depending on the
468 analyzed matrix.

469 As an example of the results furnished by PARAFAC, Fig. 6 shows the scores
470 (relative concentrations) and loadings (profiles of constituents in both modes) retrieved
471 by PARAFAC for a real assayed sample containing BTZ.

472 ** Insert Fig. 6 **

473 While the loadings allowed to identify each chemical constituent of the sample, the
474 scores corresponding to the calibrated constituent are used for building a univariate
475 calibration curve. In the studied case, constituent 1 (blue line) was assigned to BTZ. In
476 fact, the agreement between the normalized spectra of a standard BTZ solution and the
477 corresponding PARAFAC loadings of constituent 1 can be appreciated in Fig. 7.
478 Constituent 2 on the other hand (green line) constitutes an interference.

479 ** Insert Fig. 7 **

480 **4.2. Challenges of second-order fluorescence data**

481 Three important sources of problems exist in second-order fluorescent data analysis:
482 (1) deviations from the ideal trilinearity, which may occur due to the presence of inner
483 filter effects, (2) significant spectral overlapping among several sample constituents,
484 and (3) spectral profiles in one mode which are identical in all samples, due to a certain
485 physical law, reducing the selectivity in the affected mode to zero [1]. The latter
486 phenomenon may occur either between calibrated analytes or between the analyte and
487 interferences; in the latter case some algorithms cannot successfully model the data.

488

489 **4.2.1. Inner filter effects**

490 Inner filter causes deviations from trilinearity because spectra are deformed in a
491 specific manner for each chemical sample. This phenomenon can be handled by MCR-
492 ALS or PARAFAC2 only when the effect occurs in the excitation or in the emission
493 mode, but not when it occurs in both excitation and emission modes simultaneously. In
494 this latter case, only U-PLS/RBL can correctly solve this analytical problem, because of
495 its flexible structure, which allows it to account for these effects by including additional
496 latent variables in the calibration phase. In addition, if unexpected sample constituents
497 are present, the U-PLS calibration can be combined with RBL, modeling the interferent
498 contribution and achieving the second-order advantage.

499 It should be noticed that inner filter effects have been traditionally corrected by
500 sample dilution or mathematical transformations. However, this may not be possible in
501 the case of fluorescence measurements conducted in the solid surface which is
502 employed for pre-concentration purposes. It is likely that future developments of solid-
503 state luminescence measurements, where constituents are concentrated on a small area
504 of a solid membrane, will show the presently described phenomenon.

505 In 2006, the power of U-PLS/RBL was experimentally demonstrated for the first
506 time, by correctly predicting the concentration of two selected analytes (the fungicides
507 carbendazim and thiabendazole) through EEFMs read in a C18 membrane, in which the
508 analytes were retained, in the presence of unexpected species and overcoming the inner
509 filter effect [60]. Significant changes are produced by thiabendazole in both the
510 excitation and emission spectra of carbendazim. While U-PLS/RBL allowed the
511 determination of carbendazim in samples prepared with both artificial and real matrices,
512 PARAFAC showed a considerable lack of precision. Apparently, the inner-filter
513 phenomenon produced by thiabendazole cannot be modelled by PARAFAC.

514 In another 2006 report, EEFMs for samples containing mixtures of chrysene (the
515 analyte of interest), benzopyrene (which produced strong inner filter effect across the
516 useful wavelength range) and pyrene (the unexpected constituent) were successfully
517 resolved by U-PLS/RBL as the only viable alternative [61].

518

519 **4.2.2. Significant spectral overlapping among analytes and interferents**

520 Four examples, arranged in increasing order of complexity, are discussed below. In
521 the first one, one analyte and four interferents occur: galantamine (GAL), an
522 acetylcholinesterase inhibitor, was spectrofluorimetrically determined in a micellar
523 medium of sodium dodecyl sulfate through EEFMs in the presence of interferents [62].
524 Test samples constituted by GAL and the spectral interferents ibuprofen, acetyl salicylic
525 acid, phenylephrine and atropine were evaluated with PARAFAC, U-PLS/RBL and N-
526 PLS/RBL algorithms. While both PARAFAC and N-PLS rendered poor results, the U-
527 PLS/RBL predictions were in good agreement with the corresponding nominal values,
528 demonstrating the ability of this latter algorithm to successfully overcome the strong
529 spectral overlapping between the analyte and interferences.

530 The second example involves two analytes and three interferents: the fungicides
531 thiabendazole (TBZ) and fuberidazole (FBZ) were spectrofluorimetrically determined in
532 the presence of high concentrations of carbaryl, carbendazim and 1-naphthylacetic acid,
533 using an optosensor coupled to a flow-injection system [63]. The sensor was based on
534 the simultaneous retention of the analytes on C18-bonded phase placed inside a flow-
535 cell. The EEFM was read after the arrival of the analytes to the sensing zone.

536 Figure 8 shows the fluorescence excitation and emission spectra for TBZ, FBZ and
537 other agrochemicals adsorbed on the C18 solid surface, where the strong overlapping
538 among analytes and interferents is clear. U-PLS/RBL provides good predictions for
539 both TBZ and FBZ, allowing to reach selectivity using a commercial but non-selective
540 sensing support. The PARAFAC recoveries were comparably worse, especially for the
541 analyte TBZ at low concentrations. This may be ascribed to significant overlapping
542 profiles of both analytes (TBZ and FBZ) in the emission mode (Fig. 8).

543 ** Insert Fig. 8 **

544 In the third example, two analytes and fourteen interferents were studied:
545 benzo[*a*]pyrene and dibenzo[*a,h*]anthracene, the two most carcinogenic polycyclic
546 aromatic hydrocarbons (PAHs), were quantitated in a very interfering environment
547 through EEFMs measured on a nylon-membrane surface [64]. The matrices were
548 processed by applying PARAFAC and U-PLS/RBL. The superiority of U-PLS/ RBL to
549 quantify the selected PAHs in the presence of the remaining 14 US EPA (United States
550 Environmental Protection Agency) PAHs was demonstrated.

551 Finally, six PAHs, namely benzo[*a*]pyrene (BaP), dibenz[*a,h*]anthracene (DBA),
552 chrysene (CHR), benzo[*b*]fluoranthene (BbF), benzo[*k*]fluoranthene (BkF) and
553 benz[*a*]anthracene (BaA) were simultaneously quantified in the presence of other 10
554 interfering PAHs, applying second-order multivariate calibration to the EEFMs

555 obtained with a flow-through optosensor interfaced to a fast-scanning
556 spectrofluorimeter [65]. The interferences were analyzed at high concentrations, in
557 order to maximize the problem they may cause in the determination.

558 PARAFAC rendered poor results for BaP, DBA, CHR, and BaA, and this fact was
559 ascribed to a lack of selectivity for these analytes. Certainly, the significant spectral
560 overlapping among analytes and interferences appears to preclude the successful
561 decomposition of the second-order data. The ability of U-PLS/RBL to resolve highly
562 overlapped analytes was demonstrated, even in a very interfering medium. Among the
563 studied PAHs, the calculated values for BbF showed certain dispersion with respect to
564 the nominal ones, and this fact was ascribed to the presence of FLT as interference,
565 whose spectra seriously overlap with those for BbF. On the other hand, N-PLS was only
566 able to successfully predict the concentrations of BaP, BbF, and BkF. This demonstrates
567 a weaker capability of this algorithm to resolve this type of complex systems.

568

569 **4.2.3. Identical profiles for two constituents (analyte or interferent)**

570 As explained above, the U-PLS/RBL algorithm is able to resolve cases with strong
571 spectral overlapping. However, in the cases of identical profiles for analyte and
572 interferent in one mode this algorithm fails, because the RBL technique is unable to
573 distinguish the constituents. Trilinear decomposition also fails in this case. The
574 phenomenon is a special case of *linear dependency*, because a linear mathematical
575 relationship exists between the profiles of two constituents along one of the data modes.

576 These problems can be appropriately resolved by MCR-ALS. As already stated,
577 MCR-ALS decomposes an augmented data matrix, built by placing matrices for
578 different samples adjacent to each other, in such a way that the augmentation mode is

579 the one affected by the profile overlapping. As a result, the null selectivity in the
580 affected direction is recovered in the augmented mode.

581 A sample with either two responsive constituents with identical profiles in one mode
582 or the presence of an interferent with the same profile as a calibrated constituent will
583 produce a data matrix with rank one; that is, the matrix will be rank-deficient, a
584 situation also known as *rank overlap*. Analytical problems involving rank overlap
585 appear, for example, (1) when the kinetics of a reaction is followed and the reaction
586 product (unique for all sample constituents) is the responsive constituent; (2) when the
587 common mode is the luminescence time decay, corresponding to a lanthanide ion whose
588 excitation spectrum varies with the constituent that complexes the ion, (3) when the
589 emission spectrum of a species is common to all constituents, but the time evolution of
590 the signal differs and (4) when the interferent has the same profile in one of the data
591 modes as the analyte.

592 As an example of this case, we can mention the determination of three
593 fluoroquinolone antibiotics (ciprofloxacin, norfloxacin and danofloxacin) in serum in
594 the presence of the interferent salicylate [66]. The method was based on processing
595 lanthanide-sensitized excitation-time decay matrix data for their terbium (III)
596 complexes. As can be appreciated in Fig. 9, while the excitation mode shows good
597 selectivity between analyte and salicylate, in the time mode the selectivity is very poor.

598 The calculated lifetimes for the studied fluoroquinolones and salicylate are very
599 similar confirming the similarity of the corresponding decay curves.

600 MCR-ALS permitted the achievement of the second-order advantage in the presence
601 of a high degree of overlapping between the time decay profiles for the analyte and the
602 interferent complexes. Due to the presence of interactions between analytes and the
603 investigated matrix (serum), it was necessary to employ the standard addition method

604 for a successful determination. The test data matrix was subtracted from the standard
605 addition matrices, and quantitation was performed using classical external calibration
606 procedure.

607 **** Insert Fig. 9 ****

608 Figure 10 shows the MCR-ALS decomposition results for a typical test sample
609 containing ciprofloxacin and salicylate. Figure 10A displays the retrieved spectral
610 profiles, which are seen to resemble those for the terbium complexes of the analyte and
611 salicylate. Besides, Fig. 10B shows the progression of time decay profiles in the
612 corresponding standard addition study. The left sub-plot corresponds to the test sample
613 itself, while the three remaining matrices are those obtained after subtracting the test
614 sample matrix from each of the standard addition matrices (in these three sub-plots the
615 profile for the salicylate is absent).

616 **** Insert Fig. 10 ****

617 This example illustrates the success of MCR-ALS in decomposing the contributions
618 from the analyte and from the interferent, even when the time decay profile for the
619 salicylate complex is almost identical to that of the ciprofloxacin complex.

620 **4.3. Chromatography/multivariate detection matrices**

621 In chromatography, the retention factor (k), which is the degree of retention of the
622 sample constituent in the column, plays an important role in most analysis: in practice
623 analytes elute with retention factors between 1 and 20, with a peak with k equal to 0
624 indicating a constituent which does not interact with the stationary phase [67].
625 Chromatographic separations can become a difficult task when complex samples have
626 to be analyzed, because of the presence of constituents with similar retention factors.
627 Nevertheless, the use of multi-way calibration may provide a useful resource for

628 accurate analyte quantitation when complete separation is not accomplished, or new
629 constituents are present in the sample being analyzed [68].

630 The hyphenation of liquid (LC) or gas (GC) chromatography (or capillary
631 electrophoresis, CE) with spectroscopic techniques can yield second-order data which
632 combine instrumental signals built from both spectral and time domains. Popular
633 techniques are LC-DAD (UV-Visible diode array detection), LC-FSFD (fast scanning
634 fluorescence detection) or LC-MS (mass spectrometric detection). The responses are
635 thus arranged as a data matrix, where each column corresponds to a wavelength (or m/z
636 ratio) and each row corresponds to a different elution time. Figure 11 shows a typical
637 example of a second-order data matrix generated by HPLC-DAD, corresponding to the
638 fast determination of dyes in beverages [69]. As can be appreciated, the incomplete
639 separation of three dyes (plus interferents) was accomplished in ca. 1.9 min.
640 Implementation of a second-order calibration by using pure standard samples matrices
641 furnished highly accurate results when analyzing real non-alcoholic beverage samples.

642 The use of second-order multivariate algorithms has been shown to play a critical
643 role in several analytical fields, as can be gathered from a literature survey in relevant
644 analytical, chemometrics and applied journals. Specifically, an important number of
645 reports have been presented focusing on the resolution of really complex samples by
646 using liquid chromatography and exploiting the second-order advantage [69,70]. In this
647 context, extremely important issues such as reduction in analysis time and consequently
648 in costs and amount of contaminant solvents should be considered.

649 ** Insert Fig. 11 **

650 Several algorithms can be used to model this kind of data, but MCR-ALS has
651 become the choice in most of the published works. This may be due to the fact that
652 differential migration of the constituents originates dissimilarities in both retention

653 times and peak shapes (this is even more significant in CE data), leading to data without
654 the property of trilinearity (see above). MCR-ALS can efficiently solve this type of
655 problems by resorting to the mathematical resource of matrix augmentation.
656 PARAFAC2 is a variant of PARAFAC attempting to model such changes, but
657 apparently is less powerful than MCR-ALS in this regard, especially when time shifts or
658 band shapes are significant, and when potential interferences are present in test samples.
659 However, when data are conveniently pre-treated in order to alleviate the above-
660 mentioned problems, using adequate alignment strategies of elution profiles, good
661 results can be obtained by using PARAFAC or RBL-based algorithms [68,70]. For a
662 detailed description of the different pre-treatment approaches see Refs. [68,70] and a
663 recent tutorial [71].

664 As an example, we consider the development of an HPLC-FSFD method for the
665 simultaneous determination of five marker pteridines in urine samples: neopterin
666 (NEO), biopterin (BIO), pterin (PT), xanthopterin (XAN) and isoxanthopterin (ISO)
667 [72]. Figure 12 shows a chromatogram registered at $\lambda_{exc} = 272$ nm and $\lambda_{em} = 445$ nm
668 and elution times between 2.0 and 6.5 min for a urine sample, after spiking it with the
669 five analytes. The elution order was NEO, XAN, BIO, ISO, and PT, as indicated by an
670 arrow in Fig. 12. As can be appreciated in this figure, the elution profiles of XAN, BIO
671 and ISO are highly overlapped. On the other hand, an unknown peak appears between
672 the NEO and XAN peaks. In addition, several unexpected substances appear in the urine
673 matrix, making necessary to model the data with a second-order algorithm capable of
674 exploiting the second-order advantage.

675 Owing to the fact that three of the constituents present the same spectrum (linear
676 dependency, see above), these chromatographic data can be modeled following two
677 different strategies: (1) dividing the data in regions in which the spectra of target

678 analytes and interferents are different, and (2) modeling the whole data, but augmenting
679 the matrix in the direction of the linear dependency, i.e. in the direction of the spectra. It
680 should be noted that this latter strategy can be implemented because there are no elution
681 profile changes in the chromatographic direction. In addition, independently of the
682 methodology followed to process the data, a baseline correction was applied to subtract
683 the background present in the chromatograms (for more details see Ref. 72).

684 The strategy 1, which is the most usually implemented one when analyzing these
685 kind of second-order data, involves as a first step the selection of appropriate regions to
686 quantitate one analyte at a time; thus different MCR models were built to carry out the
687 analysis (shown as boxes in Fig. 12). As can be seen in this figure, the regions in which
688 the total chromatographic data were divided, in order to simplify the analysis, were the
689 following: region 1, between 2.75 and 3.12 min, in which only NEO is eluting, was
690 selected for NEO quantitation, region 2, between 3.35 and 3.58 min, in which XAN and
691 BIO are coeluting, was selected for XAN quantitation, region 3, between 3.45 and 3.74
692 min, in which XAN, BIO, and ISO are coeluting, was selected for BIO quantitation,
693 region 4, between 3.58 and 3.87 min, in which BIO and ISO are coeluting, was selected
694 for ISO quantitation, and region 5, between 3.84 and 4.20 min, in which ISO and PT are
695 coeluting, was selected for PT quantitation.

696 ** Insert Fig. 12 **

697 The usual way to process these data involves the construction of a suitable
698 augmented data matrix from which the MCR-ALS analysis retrieves one matrix
699 containing the spectral profiles for all the species present in the samples, and an
700 additional one including information which is useful to perform analyte quantitation, i.e.
701 the area under each chromatographic peak (Fig. 3). The satisfactory implementation of
702 this strategy to determine the analytes building a single MCR-ALS model (including the

703 complete experimental elution time range) requires that the spectra of the constituents
704 must be different, since the resolution is based on this fact. This strategy is not viable in
705 this example, due to the fact that NEO, BIO and PT present almost identical emission
706 spectra, leading to near zero selectivity in the spectral mode. On the other hand, as can
707 be appreciated in Fig. 12, although the time profiles of XAN, BIO and ISO are highly
708 overlapped, small differences exist in their elution time profiles, enough to allow MCR-
709 ALS resolution in the different regions. For this reason, the data were divided into time
710 regions which were processed separately, as commented above.

711 First, Fig. 13A shows the MCR-ALS spectral profiles retrieved for region number 3.
712 These spectra correspond to the three analytes present in this region (ISO, BIO and
713 XAN) and an additional one ascribed to an interfering agent. An indicative measure of
714 the quality of the MCR-ALS modeling can be attained by visual inspection of the
715 retrieved spectral profiles for the standard solution and its comparison with real analyte
716 spectra.

717 On the other hand, Fig. 13B shows the time profiles retrieved by MCR-ALS when
718 strategy 1 was applied to region 3 [matrix \mathbf{B}_{aug} in equation (2)], for two urine samples
719 spiked with the five analytes and two analyte standard samples. As mentioned above,
720 this region contains ISO, BIO and XAN, and was employed for BIO quantitation. The
721 successive boxes in Fig. 13B show: (1) a constant signal in the first two urine samples,
722 absent in the standards (this corresponds to a urine background interferent), and (2)
723 signals corresponding to ISO, BIO and XAN, all increased in the last two samples with
724 respect to the first two due to the addition of analyte standards to the urine samples. The
725 isolation of the signal which in this case was ascribed to BIO in each studied test sample
726 was used for the quantitation of this analyte. As can be observed, the interference profile
727 remains constant.

728

** Insert Fig. 13 **

729 On the other hand, the procedure of dividing into regions was avoided in strategy 2,
730 in which the complete data were used to build the augmented data matrix that allows
731 simultaneously determining all analytes. The complexity of the processing is extremely
732 reduced if the complete data are conveniently modeled following the strategy 2. In other
733 words, the resolution of each analyte by strategy 1 requires the building of one
734 particular MCR-ALS model with data belonging to the region in which it is included,
735 i.e. five MCR-ALS models are needed to quantify the five analytes present in one
736 sample (for details see Ref. 73). Strategy 2, however, involves non-standard
737 augmentation in the elution time direction, which in turns demands that elution time
738 profiles do not vary from sample to sample. This situation is however scarcely found in
739 the literature.

740

741 **5. Third-order and beyond**

742 **5.1. Four-way spectral-kinetic data**

743 Four-way data can be obtained by joining third-order data for a set of samples into a
744 four-dimensional mathematical object. It is interesting to note that only in few reports
745 four-way data have been recorded and used to develop analytical methodologies. This
746 may be attributed to our lacking of a thorough understanding of their analytical
747 advantages, or to the fact that the practical acquisition of these data arrays is still
748 difficult to implement. Hence, although in theory a large number of possible forms of
749 obtaining four-way data exist, those commonly used are the following: (1) with a single
750 instrument, EEMs (either fluorescence or phosphorescence) as a function of reaction
751 time or decay time [74] and (2) with hyphenated instruments, two-dimensional

752 chromatography with time of flight mass spectrometry (TOFMS) or diode array
753 detection (DAD), such as GC-GC-TOFMS or LC-LC-DAD, and LC-DAD as a function
754 of reaction time [75].

755 In this way, for obtaining third-order data using luminescence spectroscopy, several
756 different approaches can be found in the literature. One is based on EEM-fluorescence
757 life-time data: the first publication dates back to as early as 1990 [76], by incorporating
758 fluorescence lifetime as a third mode of information to the excitation-emission matrix
759 (EEM) using the phase modulation technique. The data were combined with the GRAM
760 algorithm, and applied to the resolution of a two constituent mixture composed of
761 benzo[*b*]fluoranthene and benzo[*k*]fluoranthene.

762 A second approach involves laser-excited time-resolved EEM fluorescence data, as
763 in the collection of fluorescence time-resolved excitation-emission data at 4.2 K *via*
764 laser-excited time-resolved Shpol'skii spectroscopy, which has been recently proposed
765 for the resolution of mixtures of 15 polycyclic aromatic hydrocarbons [77]. These data
766 could be successfully processed using PARAFAC and U-PLS/RTL. Figure 14 shows an
767 schematic representation of how the EEMs were collected following the decay time.

768 ** Insert Fig. 14 **

769 A third alternative is based on laser-excited time-resolved EEM phosphorescence
770 data, using instrumental data generated from Shpol'skii matrices at liquid helium
771 temperature. In this case, the third-order data arrays, consisting of excitation modulated
772 wavelength-time matrices, were collected with the aid of a cryogenic fiber-optic probe,
773 a tunable dye laser, and a multichannel system for phosphorescence detection, and
774 applied to the analysis of 2,3,7,8-tetrachloro-dibenzo-para-dioxin [78].

775 Another possibility is to measure EEM-reaction time data. An example is presented
776 in which four-way fluorescence data were recorded by following the kinetic evolution

777 of EEMs, analyzed by PARAFAC and TLLS/RTL. They were applied to the
778 simultaneous determination of the constituents of the anticancer combination of
779 methotrexate and leucovorin in human urine [74]. Both analytes were converted into
780 highly fluorescent products by oxidation with potassium permanganate, and the kinetic
781 of the reaction was continuously monitored by recording full EEMs of the samples at
782 different reaction times. A fast scanning spectrofluorimeter was used for the acquisition
783 of a complete EEM in 12 seconds at a wavelength scanning speed of 24,000 nm/min.
784 The emission spectra were recorded from 335 nm to 490 nm at 5 nm intervals, exciting
785 from 255 nm to 315 nm at 6 nm intervals. Ten successive EEMs were measured at 72
786 seconds intervals, in order to follow the fluorescence kinetic evolution of the mixture
787 constituents (Fig. 15). The excitation, emission and kinetic time profiles recovered by
788 both chemometric techniques are in good agreement with experimental observations.

789 Similarly, the evolution of on-line photochemically induced excitation-emission
790 matrices with the irradiation time, allowed the determination of folic acid (FA), and its
791 two main metabolites, tetrahydrofolic acid (THF) and 5-methyltetrahydrofolic acid (5-
792 MTF), in serum samples (Fig. 16). In this figure it can be appreciated that FA and THF
793 show an initially weak fluorescence, and that the intensity of FA and THF considerably
794 increases as a function of reaction time, but at different rates. The strongly fluorescent
795 photoproducts, after irradiation of FA and THF with UV light in acidic medium, have
796 identical excitation and emission wavelengths, 270 nm and 445 nm, respectively. In
797 contrast, the strong native fluorescence of 5-MTF decreased with reaction time. The
798 method achieves selectivity from the different rates at which the corresponding
799 photoproducts are formed and degraded, as a discriminatory parameter, allowing the
800 successful determination of the three constituents by using a combination of U-PLS or
801 N-PLS with RTL [79].

802 ** Insert Fig. 15 **

803 ** Insert Fig. 16 **

804 **5.2. Chromatography/multivariate detection**

805 One common example of four-way/third-order data involving chromatography is
806 comprehensive two-dimensional gas chromatography followed by mass spectrometric
807 detection (GC-GC-MS). As mentioned in Section 3.2., multi-way data of
808 chromatographic origin usually lose multi-linearity in the time mode because of elution
809 time changes from sample to sample. Consequently, in order to apply MCR-ALS to
810 these data, it is necessary to first unfold the original third-order array for each sample
811 ($\text{time1} \times \text{time2} \times \text{mass/charge ratio}$) into a matrix (unfolded time \times mass/charge ratio
812 second-order data). These matrices are then arranged into a bilinear augmented matrix
813 and processed as shown in Fig. 5 and commented in the corresponding section.

814 Tauler et al. reported the resolution and quantitation of mixtures of polycyclic
815 aromatic hydrocarbons (PAH) in heavy fuel oil samples by MCR-ALS modelling of
816 data obtained with $\text{GC} \times \text{GC-TOFMS}$ [44]. Different three-way data corresponding to
817 pure PAH standards and real samples were transformed into matrices and submitted to
818 MCR-ALS analysis of the augmented matrix. This procedure allowed to obtain
819 component elution profiles in the two chromatographic dimensions and their pure mass
820 spectra.

821 The general procedure followed in this work for organizing the data and their
822 subsequent MCR-ALS analysis can be appreciated in Fig. 5. The samples were injected
823 into the first column, the eluted compounds were then pre-concentrated in the
824 modulator, and then re-injected into the second column after a modulation period. Thus,
825 the entire first column chromatogram was sliced into a series of high-speed short
826 secondary chromatograms of a length equal to P_M , which were continuously recorded

827 by the TOFMS detector. Every slice produced a data matrix \mathbf{X} of size $(J \times K)$, where J is
828 the number of collected data points (elution times, 2t_R) in the second column, and K is
829 the number of m/z values. The slices obtained at the different elution times in the first
830 mode (1t_R) were then stacked to form an augmented matrix of size $(LJ \times K)$ as shown in
831 Fig. 5, in which L is the number of collected data points taken from the first column (L).
832 Here LJ represents the product of the data points taken from the first column by the data
833 points taken from the second column (${}^2t_R \times {}^1t_R$). After this phase, a super-augmented data
834 matrix $\mathbf{D}_{\text{sup-aug}}$ of size $[LJ(I+1) \times K]$ was built, in which I is the number of standards and
835 "1" represents the unknown sample being analyzed. Owing to the fact that the number
836 of m/z values is equal for all slices and for all chromatographic runs, the column-wise
837 data arrangement shown in $\mathbf{D}_{\text{sup-aug}}$ is very flexible and adequate to bilinear modeling
838 requirements. For example, the number of chromatographic ranges analyzed can be
839 different in each slice, and the presence of time shifts among different slices and among
840 different samples do not destroy the bilinear model assumption associated to the
841 column-wise data augmentation strategy. On the other hand, each slice in the super-
842 augmented data matrix can have different numbers of rows (elution times in the first and
843 second modes). This is a very flexible property of matrix augmentation, which adapts
844 very well to GC \times GC-TOFMS data, because of ubiquitous elution time peak shifts of the
845 eluted components from slice-to-slice (in the second column) and from sample-to-
846 sample (in the first and second columns). Furthermore, with this super-augmented data
847 arrangement, there is no limitation in the number of included sub-matrices (number of
848 slices and samples). For quantitative analysis, it is possible to include the second mode
849 slices taken from the first column for the different standard mixtures and unknown
850 samples and their replicates in the same column-wise super-augmented data matrix, and
851 perform their simultaneous analysis in one shot [44].

852 MCR-ALS analysis yields a super-augmented concentration matrix containing the
853 pure second dimension elution profiles for different slices of different samples ($\mathbf{C}_{\text{sup-aug}}$),
854 a matrix of pure mass spectra profiles (\mathbf{S}^T) for the N components, and a residual matrix
855 ($\mathbf{E}_{\text{sup-aug}}$) containing noise and unresolved background (Fig. 5). The $\mathbf{C}_{\text{sup-aug}}$ matrix
856 contains the second mode elution profiles in all $[L(I+1) \times K]$ slices for the N resolved
857 components. The single pure mass spectral matrix \mathbf{S}^T ($N \times K$) can be used for the
858 identification of the resolved components. On the other hand, to get the first mode
859 elution profiles for every component in each sample analyzed, the areas under the
860 resolved second mode elution profiles corresponding to one sample are used (see Fig.
861 5). In this way, a matrix of size $(L \times N)$ for every sample is obtained. In addition, areas
862 under the resolved second dimension elution profiles can be used for quantitative
863 purposes.

864 In another very recently reported application, a four-way multivariate calibration
865 approach based on the combination of HPLC data and third-order algorithms has been
866 described for the first time [80]. In this case, each sample was injected into the
867 chromatograph eight times, each time exciting with a different excitation wavelength,
868 and the emission spectra were recorded along the full chromatogram. The third-order
869 data thus obtained were joined into a four-way data array, which was subsequently
870 analyzed with PARAFAC, U-PLS/RTL and N-PLS/RTL, because no substantial
871 changes in elution profiles were detected from sample to sample. The best algorithm to
872 perform the multi-way calibration was U-PLS/RTL.

873

874 **5.3. Five-way data**

875 Five-way data can be obtained by joining fourth-order data for a sample set. The
876 algorithm U-PLS/RQL has been developed as a new latent structured algorithm for the
877 processing of these instrumental data. In order to check its analytical predictive ability,
878 fluorescence excitation-emission-kinetic-pH data were measured and processed. The
879 concentration of the fluorescent pesticide carbaryl was determined in the presence of
880 uncalibrated interferents, in the first reported example of fourth-order multivariate
881 calibration.

882 **** Insert Fig. 17 ****

883 The hydrolysis of the analyte was followed at different pH values using a fast-
884 scanning spectrofluorimeter, recording the excitation-emission fluorescence matrices
885 during its evolution to produce 1-naphthol, which does also emit fluorescence (Fig. 17)
886 [47].

887

888 **6. Multi-way analytical figures of merit**

889 The development of new multi-way analytical methods demands to be able to
890 estimate the corresponding analytical figures of merit, in order to compare with
891 previously existing methodologies and to report detection capabilities and other
892 important features [10]. In this context, the sensitivity is a crucial parameter, because:
893 (1) it allows proper comparison among different methods, (2) it permits the estimation
894 of other figures of merit, such as the response-independent analytical sensitivity [81],
895 and (3) it is needed to compute prediction uncertainties and detection capabilities [82].

896 Pertinent questions in this area are the following: (1) is there any conceptual
897 difference between classical univariate figures of merit and their multi-way analogues?
898 and (2) can multi-way figures of merit be estimated for all calibration methodologies
899 and models? The answers to these important questions are, fortunately, no and yes,
900 respectively. Below we present a brief discussion connecting the classical approach with
901 the advanced field of multi-way figures of merit.

902 According to the International Union of Pure and Applied Chemistry (IUPAC), the
903 sensitivity is well-defined in some analytical calibration [83-85], such as univariate
904 calibration, where it is the change in the instrument response for a unit change in the
905 concentration of the analyte of interest [83]. The slope of the calibration graph is in this
906 case a convenient measure of the sensitivity [83]. For first-order multivariate
907 calibration, this definition has been generalized employing an intuitive analogy between
908 instrumental signal and a so-called net analyte signal (NAS) [86]. The NAS is defined
909 as the portion of the overall signal which can be uniquely ascribed to the analyte [86,87]
910 and is completely general and applicable to all first-order calibration methods, including
911 the popular partial least-squares (PLS) algorithm [87]. Mathematically, the NAS for a
912 given analyte is defined as the projection of the sample signal orthogonal to the space
913 spanned by the interferent agents (Fig. 18A). A plot of the length of NAS vectors as a
914 function of the analyte concentrations should be linear, the slope being the length of the
915 NAS vector at unit concentration, which is a good measure of the first-order sensitivity.

916 In the case of second-order multivariate calibration, there have been several
917 proposals for estimating the sensitivity [88-90]. Some of them were based in extensions
918 of the NAS concept, although difficulties appear because: (1) there are different NAS
919 definitions, and it is difficult to understand their true meanings [91-93] and (2)
920 extrapolation to orders higher than two leads to a significantly underestimation of true

921 sensitivity values [94]. We discuss as an example the most general second-order
 922 PARAFAC sensitivity, whose expression has been shown to be [90]:

$$923 \quad \text{SEN} = s_n \{ [(\mathbf{B}_{\text{cal}}^T (\mathbf{I} - \mathbf{B}_{\text{unx}} \mathbf{B}_{\text{unx}}^+) \mathbf{B}_{\text{cal}}) * (\mathbf{C}_{\text{cal}}^T (\mathbf{I} - \mathbf{C}_{\text{unx}} \mathbf{C}_{\text{unx}}^+) \mathbf{C}_{\text{cal}})]^{-1} \}_{nn}^{-1/2} \quad (4)$$

924 where s_n is the slope of the PARAFAC univariate plot, the index n identifies the analyte,
 925 the symbol '*' is the Hadamard matrix product, the subscript 'nn' indicates the (n,n)
 926 diagonal element of a matrix, the matrices \mathbf{B} and \mathbf{C} collect the loadings, the subscripts
 927 'cal' and 'unx' indicate calibration and unexpected respectively, and the symbol '+'
 928 implies the pseudo-inverse of a matrix.

929 A recent alternative definition is based on the concepts of input and output noise in a
 930 given system: the sensitivity (SEN) measures the ratio of output noise to input noise
 931 [95,96]. This approach allowed to develop sensitivity expressions for multi-way
 932 calibration based on PARAFAC [94], MCR-ALS [97] and PLS/RML (RML indicates
 933 residual multi-linearization, and includes RBL, RTL, RQL, etc.) [98]. Further work is
 934 needed, however, to validate these expressions and to include all of them into a
 935 generalized conceptual scheme.

936 It is important to notice, however, that the estimation of figures of merit is included
 937 in some of the available software packages for multi-way calibration, and thus by
 938 employing this software the analyst has access to all analytical figures to be reported
 939 along with the concentration of the analyte of interest: MVC2 and MVC3 for second-
 940 and third-order calibration respectively (see Table 2) [51,52].

941 **** Insert Fig. 18 ****

942 The sensitivity is important because it allows estimating additional figures of merit
 943 [10], such as the uncertainty in predicted concentrations. The best known approximation
 944 to concentration variance is the following expression:

$$945 \quad \sigma_y^2 = \text{SEN}^{-2} \sigma_x^2 + h \text{SEN}^{-2} \sigma_x^2 + h \sigma_{\text{ycal}}^2 \quad (5)$$

946 where SEN is the sensitivity, σ_x^2 the variance in instrumental signals, h the sample
947 leverage and $\sigma_{y_{\text{cal}}}^2$ the variance in calibration concentrations. The three terms in
948 equation (5) correspond to the propagation of uncertainties from: instrumental signals in
949 the test sample, instrumental signals in the calibration, and calibration concentrations.
950 The last two terms are scaled by the sample leverage h , a dimensionless parameter
951 measuring the relative position of the sample in the calibration space.

952 The limit of detection (LOD) and quantitation (LOQ) are additional important figures
953 of merit. They can be estimated based on IUPAC's recommendations on the so-called
954 type I and II errors [82]. It is first required to define a critical concentration value, which
955 is the level for the detection decision, involving a certain risk of type I errors (α is the
956 probability of false positive). In this way, the decision “detected” or “not detected” is
957 made by comparison of the estimated quantity (\hat{L}) with the critical value (L_C) of the
958 respective distribution, such that the probability of exceeding L_C is no greater than α if
959 the analyte is absent. The limit of detection is defined as the concentration (LOD) for
960 which the risk of Type II errors has a probability β (β is the probability of false
961 negative) given L_C or α . Both α and β are assigned reasonable values depending on the
962 specific analytical application. If it can be assumed that the concentration uncertainty at
963 zero analyte concentration is close to that at the LOD level, the latter can be
964 approximated by $(t_{\alpha,v} + t_{\beta,v}) \sigma_0$, where σ_0 is the prediction uncertainty for a blank
965 sample, and $t_{\alpha,v}$ and $t_{\beta,v}$ are the t -coefficients for probabilities α and β with v degrees of
966 freedom. For 95% probabilities of coverage against both type I and II errors and a large
967 number of degrees of freedom, the limit of detection can be estimated as the product of
968 a specific coefficient and the standard deviation of the blank, when the uncertainty in

969 the mean (expected) value of the blank is negligible, and α and β are each equal 0.05,
970 and the estimated value is normally distributed with known, constant variance.

$$971 \quad \text{LOD} = 3.3 \sigma_0 \quad (6)$$

972 The limit of quantitation, in turn, is estimated as:

$$973 \quad \text{LOQ} = 10 \sigma_0 \quad (7)$$

974 which ensures a maximum of 10% relative error in prediction.

975 We now describe in detail a work which nicely illustrates the consequences of the
976 interplay among the different terms in the LOD expression. The main purpose of this
977 description is to alert readers on the fact that the significant experimental effort required
978 to achieve larger sensitivity may not be directly translated into a proportional decrease
979 in limit of detection. The work in question discusses the determination of a fluorescent
980 pesticide in the presence of other fluorescent interferents in water samples, which has
981 been accomplished using multi-way methodologies of increasing order [47]. The most
982 complex work involved the measurement of five-way data for a group of samples, i.e.,
983 fourth-order data based on the kinetic evolution of excitation-emission fluorescence
984 matrices as a function of pH as stated above [47]. The four data modes for each sample
985 were excitation wavelength, emission wavelength, reaction time and pH. It is important
986 to notice that these data are not quadri-linear, since the rate constant for the hydrolysis
987 of the analyte is pH-dependent, and hence two data modes are highly correlated. This
988 precludes, in principle, the use of PARAFAC, which requires quadri-linearity in the
989 data for each sample. However, concatenation of the reaction time and pH modes into a
990 single combined mode does allow for quadri-linear PARAFAC decomposition of a
991 four-way array whose modes are sample, excitation, emission and the concatenation of
992 pH and time.

993 Fourth-order data provide the second-order advantage and in principle increased
994 sensitivity and improved detection capability. However, the second-order advantage
995 could also be achieved by third-order data (measuring the same data set at a fixed pH, or
996 at a fixed reaction time), or even by second-order data (measuring the excitation-
997 emission matrices at a fixed reaction time and pH values). What is exactly gained in
998 going from second- to third- to fourth-order data can be estimated from the calculation
999 of the analytical figures of merit for each alternative. It is not the purpose of this work to
1000 comprehensively cover all the possibilities, which are too many, and hence we limit to
1001 only the available alternative measurement schemes.

1002 Since the data set in its original structure is fourth-order and is not quadri-linear, the
1003 best data processing algorithm is U-PLS/RQL, because of its inherent flexibility
1004 towards non multi-linear data. The sensitivity can be precisely calculated for three data
1005 orders: (1) second-order fixing reaction time and pH, (2) third-order fixing pH, and (3)
1006 fourth-order [98]. The result is shown in Fig. 18B, where a clear gain in sensitivity is
1007 detected.

1008 However, Fig. 18C shows the progression of the LOD value. As can be seen, the
1009 LOD seems to level off at third-order, this is because the contribution of the remaining
1010 terms in equation (5), which depend not only on the sensitivity but also on the sample
1011 leverage. The propagated calibration uncertainty, particularly the one on concentrations,
1012 which does not depend on the value of SEN, is mainly responsible for the leveling off
1013 action on increasing the data order. These considerations should become useful when
1014 planning multi-way experiments, since it is clear that an increase in sensitivity does not
1015 lead, per se, to a concomitantly decrease in LOD.

1016 Finally, two other approaches to estimating LOD and LOQ should be mentioned.
1017 One of them involves the consideration of the univariate calibration plot as a true

1018 single-component calibration, and estimation of these limits directly by the classical
1019 univariate approach. This has been done in the framework of PARAFAC and MCR-
1020 ALS [99-101]. In ref. [102] it is mathematically derived that the decision and detection
1021 limits as defined by IUPAC are the same if the regression concentration calculated
1022 versus nominal concentration is used instead of the signal versus nominal concentration.
1023 In this new equivalent formulation the definitions are applicable to any calibration
1024 model because, once built, it allows to compute the calculated concentration.
1025 Applications of this procedure can be seen in [103]. Further work is required to
1026 investigate the effect of potential interferents and the second-order advantage in the
1027 latter formulation.

1028

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1040 **References**

- [1] G.M. Escandar, N.M. Faber, H.C. Goicoechea, A. Muñoz de la Peña, A.C. Olivieri, R.J. Poppi, *Trends Anal. Chem.* 26 (2007) 752-765.
- [2] A.C. Olivieri, G.M. Escandar, A. Muñoz de la Peña, *Trends Anal. Chem.* 30 (2011) 607-617.
- [3] S. Mas, A. de Juan, R. Tauler, A.C. Olivieri, G.M. Escandar, *Talanta* 80 (2010) 1052-1067.
- [4] E. Sanchez, B.R. Kowalski, *J. Chemom.* 2 (1988) 265-280.
- [5] C.-N. Ho, G.G. Christian, E.R. Davidson, *Anal. Chem.* 50 (1978) 1108-1113.
- [6] J.A. Arancibia, P.C. Damiani, G.M. Escandar, G.A. Ibañez, A.C. Olivieri, *J. Chromatogr. B* 910 (2012) 22-30.
- [7] M.C. Ortiz, L. Sarabia, *J. Chromatogr. A* 1158 (2007) 94-110.
- [8] V. Gómez, M.P. Callao, *Anal. Chim. Acta* 627 (2008) 169-183.
- [9] C. Ruckebusch, L. Blanchet *Anal. Chim. Acta* 765 (2013) 28-36
- [10] A.C. Olivieri, N.M. Faber, *Validation and Error*, in S. Brown, R. Tauler, B. Walczak (Eds.), *Comprehensive Chemometrics*, Elsevier, Amsterdam, 2009, Vol. 3, pp. 91-120.
- [11] W.E. Van der Linden, *Pure Appl. Chem.* 61 (1989) 91-95.
- [12] K.S. Booksh, B.R. Kowalski, *Anal. Chem.* 66 (1994) 782A-791A
- [13] J. Ferré, N. M. Faber, *Chemom. Intell. Lab. Syst.* 69 (2003) 123-126.
- [14] N. M. Faber, *Chemom. Intell. Lab. Syst.* 50 (2000) 107-114.
- [15] R. Bro, *Chemom. Intell. Lab. Syst.* 38 (1997) 149-171.
- [16] A. Smilde, R. Bro, P. Geladi, *Multi-way analysis: applications in the chemical sciences*, Wiley, Chichester, 2004.

- [17] R. Bro, *Crit. Rev. Anal. Chem.* 36 (2006) 279-293.
- [18] P. M. Kroonenberg, *Applied multiway data analysis*, Wiley, Chichester, 2008.
- [19] P.A. Paatero, *Chemom. Intell. Lab. Syst.* 38 (1997) 223-242.
- [20] Z.P. Chen, H. L. Wu, J. H. Jiang, Y. Li, R. Q. Yu, *Chemom. Intell. Lab. Syst.* 52 (2000) 75-86.
- [21] A.L. Xia, H.L. Wu, D. M. Fang, Y.J. Ding, L.Q. Hu, R.Q. Yu, *J. Chemom.* 19 (2005) 65-76.
- [22] E. Sanchez, B.R. Kowalski, *Anal. Chem.* 58 (1986) 496-499.
- [23] E. Sanchez, B.R. Kowalski, *J. Chemom.* 4 (1990) 29-45.
- [24] M. Linder, R. Sundberg, *J. Chemom.* 16 (2002) 12-27.
- [25] A. de Juan, R. Tauler, *J. Chemom.* 15 (2001) 749-772.
- [26] R. Tauler, *Chemom. Intell. Lab. Syst.* 30 (1995) 133-146.
- [27] A. de Juan, R. Tauler, *Crit. Rev. Anal. Chem.* 36 (2006) 163-176.
- [28] A. de Juan, R. Tauler, *Anal. Chim. Acta* 500 (2003) 195-210.
- [29] R. Tauler, M. Maeder, A. de Juan, *Multiset Data Analysis: Extended Multivariate Curve Resolution*, in S. Brown, R. Tauler, R. Walczak (Eds.), *Comprehensive Chemometrics*, Elsevier, Amsterdam, 2009, Vol. 2, pp. 473-505.
- [30] W. Windig, J. Guilment, *Anal. Chem.* 63 (1991) 1425-1432.
- [31] M.C. Antunes, J.E.J. Simao, A.C. Duarte, R. Tauler, *Analyst* 127 (2002) 809-817.
- [32] H.A. L. Kiers, J.M.F. ten Berge, R. Bro, *J. Chemom.* 13 (1999) 275-294.
- [33] J.M. Amigo, T. Skov, J. Coello, S. MasPOCH, R. Bro, *Trends Anal. Chem.* 27 (2008) 714-725.
- [34] R. Bro, C.A. Andersson, H.A. L. Kiers, *J. Chemom.* 13 (1999) 295-309.
- [35] R. Tauler, I. Marqués, E. Casassas, *J. Chemom.* 12 (1998) 55-75.

- [36] B.E. Wilson, W. Lindberg, B.R. Kowalski, *J. Am. Chem. Soc.* 111 (1989) 3797-3804.
- [37] S. Wold, P. Geladi, K. Esbensen, J. Øhman, *J. Chemom.* 1 (1987) 41-56.
- [38] R. Bro, *J. Chemom.* 10 (1996) 47-61.
- [39] J. Öhman, P. Geladi, S. Wold, *J. Chemom.* 4 (1990) 79-90.
- [40] A.C. Olivieri, *J. Chemom.* 19 (2005) 253-265.
- [41] A.L. Xia, H.L. Wu, S.F. Li, S. H. Zhu, L.Q. Hu, R.Q. Yu, *J. Chemom.* 21 (2007) 133-144.
- [42] H.Y. Fu, H.L. Wu, Y.J. Yu, L.L. Yu, S.R. Zhang, J.F. Nie, S.F. Li, R.Q. Yu, *J. Chemom.* 25 (2011) 408-429.
- [43] H.P. Bailey, S.C. Rutan, *Chemom. Intell. Lab. Syst.* 106 (2011) 131-141.
- [44] H. Parastar, J.R. Radovic, M. Jalali-Heravi, S. Diez, J.M. Bayona, R. Tauler, *Anal. Chem.* 83 (2011) 9289-9297.
- [45] P.C. Damiani, I. Durán Merás, A.G García Reiriz, A. Jimenez Girón, A. Muñoz de la Peña, A.C. Olivieri, *Anal. Chem.* 76 (2007) 6949-6958.
- [46] J.A. Arancibia, A.C. Olivieri, D. Bohoyo Gil, A. Espinosa Mansilla, I. Durán Merás, A. Muñoz de la Peña, *Chemom. Intell. Lab. Syst.* 80 (2006) 77-86.
- [47] R.M. Maggio, A. Muñoz de la Peña, A.C. Olivieri, *Chem. Intell. Lab. Syst.* 109 (2011) 178-185.
- [48] MATLAB, The Mathworks, Inc., Natick, Massachusetts, USA.
- [49] J. Jaumot, R. Gargallo, A. de Juan, R. Tauler, *Chemom. Intell. Lab. Syst.* 76 (2005) 101-110.
- [50] P.J. Gemperline, E. Cash, *Anal. Chem.* 75 (2003) 4236-4243.
- [51] A.C. Olivieri, H.L. Wu, R.Q. Yu, *Chemom. Intell. Lab. Syst.* 96 (2009) 246-251.

- [52] A.C. Olivieri, H.L. Wu, R.Q. Yu, *Chemom. Intell. Lab. Syst.* 116 (2012) 9-16.
- [53] A. Espinosa-Mansilla, A. Muñoz de la Peña, H.C. Goicoechea, A.C. Olivieri, *Appl. Spectrosc.* 58 (2004) 83-90.
- [54] H.C. Goicoechea, A.C. Olivieri, *Appl. Spectrosc.* 59 (2005) 926-933.
- [55] A. Muñoz de la Peña, N. Mora Díez, D. Bohoyo Gil, E. Cano Carranza, J. *Fluoresc.* 19 (2009) 345-352.
- [56] A. Muñoz de la Peña, A. Espinosa Mansilla, D. González Gómez, A.C. Olivieri, H.C. Goicoechea, *Anal. Chem.* 75 (2003) 2640-2646.
- [57] A. Jimenez Girón, I. Durán Merás, A. Muñoz de la Peña, A. Espinosa Mansilla, F. Cañada Cañada, A.C. Olivieri, *Anal. Bioanal. Chem.* 391 (2008) 827-835.
- [58] J.A. Porini, G.M. Escandar, *Anal. Meth.* 3 (2011) 1494-1500.
- [59] <http://www.models.kvl.dk/~pih/parafac/chap2parafac.htm>
- [60] G.N. Piccirilli, G.M. Escandar, *Analyst* 131 (2006) 1012-1020.
- [61] D. Bohoyo Gil, A. Muñoz de la Peña, J.A. Arancibia, G.M. Escandar, A.C. Olivieri, *Anal. Chem.* 78 (2006) 8051-8058.
- [62] M.J. Culzoni, R.Q. Aucelio, G. M. Escandar, *Talanta* 82 (2010) 325-332.
- [63] G.N. Piccirilli, G.M. Escandar, *Analyst* 135 (2010) 1299-1308.
- [64] S.A. Bortolato, J.A. Arancibia, G.M. Escandar, *Anal. Chem.* 80 (2008) 8276-8286.
- [65] S.A. Bortolato, J.A. Arancibia, G.M. Escandar, *Environ. Sci. Technol.* 45 (2011) 1513-1520.
- [66] V.A. Lozano, R. Tauler, G.A. Ibañez, A.C. Olivieri, *Talanta* 77 (2009) 1715-1723.

- [67] M.W. Dong, *Modern HPLC for Practicing Scientists*, Wiley, Synomics Pharmaceutical Services, LLC, Wareham, Massachusetts, 2006.
- [68] M. Daszykowski, B. Walczak, *Trends Anal. Chem.* 25 (2006) 1081-1096.
- [69] M.J. Culzoni, A.V. Schenone, N.E. Llamas, M. Garrido, M.S. Di Nezio, B.S. Fernández Band, H.C. Goicoechea, *J. Chromatogr. A* 1216 (2009) 7063-7070.
- [70] H.C. Goicoechea, M.J. Culzoni, M.D. Gil García, M. Martínez Galera, *Talanta* 83 (2011) 1198-1107.
- [71] T.G. Bloemberg, J. Gerretzen, A. Lunshof, R. Wehrens, L.M.C. Buydens, *Anal. Chim. Acta* 781 (2013) 14-32.
- [72] A. Mancha de Llanos, M.M. De Zan, M.J. Culzoni, A. Espinosa Mansilla, F. Cañada Cañada, A. Muñoz de la Peña, H.C. Goicoechea, *Anal. Bioanal. Chem.* 399 (2011) 2123-2135.
- [73] M.J. Culzoni, A. Mancha de Llanos, M.M. De Zan, A. Espinosa Mansilla, F. Cañada Cañada, A. Muñoz de la Peña, H.C. Goicoechea, *Talanta* 85 (2011) 2368-2374.
- [74] A.C. Olivieri, J.A. Arancibia, A. Muñoz de la Peña, I. Durán Merás, A. Espinosa Mansilla, *Anal. Chem.* 76 (2004) 5657-5666.
- [75] A.E. Sinha, B.J. Prazen, R.E. Synovec, *Anal. Bioanal. Chem.* 378 (2004) 1948-1951.
- [76] D.S. Burdick, X.M. Tu, L.B. McGown, D.W. Millican, *J. Chemom.* 4 (1990) 15-28.
- [77] H.C. Goicoechea, S. Yu, A.F.T. Moore, A.D. Campligia, *Talanta* 101 (2012) 330-336.

- [78] H.C. Goicoechea, S. Yu, A.C. Olivieri, A.D. Campligia, *Anal. Chem.* 77 (2005) 2608-2616.
- [79] A. Jiménez Girón, I. Durán Merás, A. Espinosa Mansilla, A. Muñoz de la Peña, F. Cañada Cañada, A. C. Olivieri, *Anal. Chim. Acta* 622 (2008) 94-103
- [80] V. Lozano, A. Muñoz de la Peña, I. Durán Merás, A. Espinosa Mansilla, G.M. Escandar, *Chemom. Intell. Lab. Syst.* 125 (2013) 121-131.
- [81] L. Cuadros Rodríguez, A.M. García Campaña, C. Jiménez Linares, M. Román Ceba, *Anal. Lett.* 26 (1993) 1243-1258.
- [82] L.A. Currie, *Pure Appl. Chem.* 67 (1995) 1699-1723.
- [83] K. Danzer, L.A. Currie, *Pure Appl. Chem.* 70 (1998) 993-1014.
- [84] K. Danzer, M. Otto, L.A. Currie, *Pure Appl. Chem.* 76 (2004) 1215-1225.
- [85] A.C. Olivieri, N.M. Faber, J. Ferré, R. Boqué, J. Kalivas, H. Mark, *Pure Appl. Chem.* 78 (2006) 633-661.
- [86] A. Lorber, *Anal. Chem.* 58 (1986) 1167-1172.
- [87] A. Lorber, K. Faber, B.R. Kowalski, *Anal. Chem.* 69 (1997) 1620-1626.
- [88] C.N. Ho, G.D. Christian, E.R. Davidson, *Anal. Chem.* 52 (1980) 1071-1079.
- [89] N.J. Messick, J.H. Kalivas, P.M. Lang, *Anal. Chem.* 68 (1996) 572-1579.
- [90] A.C. Olivieri, N.M. Faber, *J. Chemom.* 19 (2005) 583-592.
- [91] K. Faber, A. Lorber, B.R. Kowalski, *J. Chemom.* 11(1997) 419-461.
- [92] A.C. Olivieri, *Anal. Chem.* 77 (2005) 4936-4946.
- [93] A.C. Olivieri, *Anal. Chem.* 80 (2008) 5713-5720.
- [94] A.C. Olivieri, N.M. Faber, *Anal. Chem.* 84 (2012) 186-193.
- [95] A. Saltelli, M. Ratto, S. Tarantola, F. Campolongo, *Chem. Rev.* 105 (2005) 2811-2828.

- [96] N.M. Faber, J. Ferré, R. Boqué, J.H. Kalivas, *Chemom. Intell. Lab. Syst.* 63 (2002) 107-116.
- [97] C. Bauza, G.A. Ibañez, R. Tauler, A.C. Olivieri, *Anal Chem* 84 (2012) 8697-8706.
- [98] F.A. Allegrini, A.C. Olivieri, *Anal. Chem.* 84 (2012) 10823-10830.
- [99] J. Saurina, C. Leal, R. Compañó, M. Granados, M. Dolors Prat, R. Tauler, *Anal. Chim. Acta* 432 (2001) 241-251.
- [100] M.J. Rodríguez-Cuesta, R. Boqué, F.X. Rius, *Anal. Chim. Acta* 491 (2003) 47-57.
- [101] M.J. Rodríguez-Cuesta, R. Boqué, F.X. Rius, J.L. Martínez Vidal, A. Garrido Frenich, *Chemom. Intell. Lab. Syst.* 77 (2005) 251-260.
- [102] M.C. Ortiz, L.A. Sarabia, A. Herrero, M.S. Sánchez, M.B. Sanza, M.E. Rueda, D. Giménez, M.E. Meléndez, *Chemom. Intell. Lab. Syst.* 69 (2003) 21-33.
- [103] M.C. Ortiz, L.A. Sarabia, M.S. Sánchez, *Anal. Chim. Acta* 674 (2010) 123-142.

Table 1. Different arrays that can be obtained for a single sample and for a set of samples.











Data order	Array		Calibration		
	One sample	A sample set			
Zero	Scalar		One-way		Univariate
First	Vector		Two-way		Multivariate
Second	Matrix		Three-way		Multi-way
Third	Three-way		Four-way		Multi-way
Fourth	Four-way		Five-way		Multi-way

Table 2. Free software for multi-way data processing.

Name	Algorithm(s)	Web page ^a
The N-way toolbox	PARAFAC PARAFAC2 GRAM DTLD U-PLS N-PLS	http://www.models.life.ku.dk/nwaytoolbox/download
MCR graphical interface	MCR-ALS	http://www.mcrals.info/
GUIPRO graphical interface	MCR-ALS	http://personal.ecu.edu/gemperlinep
MVC2 graphical interface	PARAFAC APTLD SWATLD BLLS/RBL U-PLS/RBL N-PLS/RBL	www.iquir-conicet.gov.ar/descargas/mvc2.rar
MVC3 graphical interface	PARAFAC APQLD AWRCQLD TLLS/RTL U-PLS/RTL N-PLS/RTL	www.iquir-conicet.gov.ar/descargas/mvc3.rar

^a All pages were accessed in March 2013.

Figures and captions

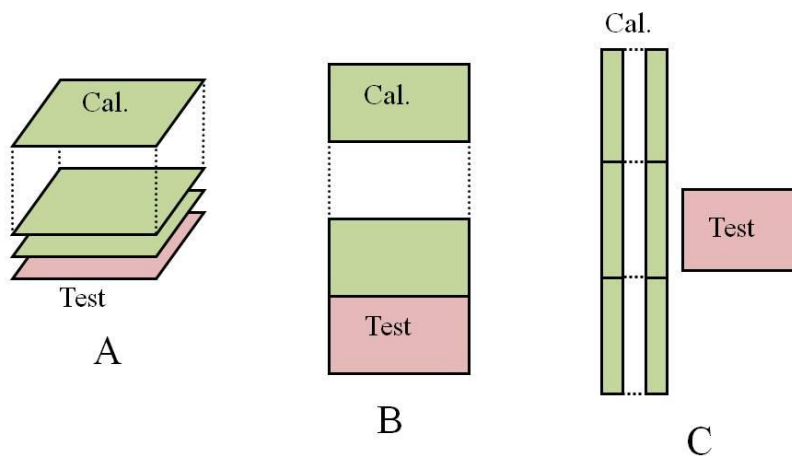


Fig. 1. Schematic representation of the way in which three different algorithms may organize second-order data. A) Joining the data matrices (calibration and test) into a three-way array. B) Placing data matrices adjacent to each other and creating a so-called augmented data matrix. C) Unfolding the calibration matrices into vectors, while keeping the matrix structure of the test sample data.

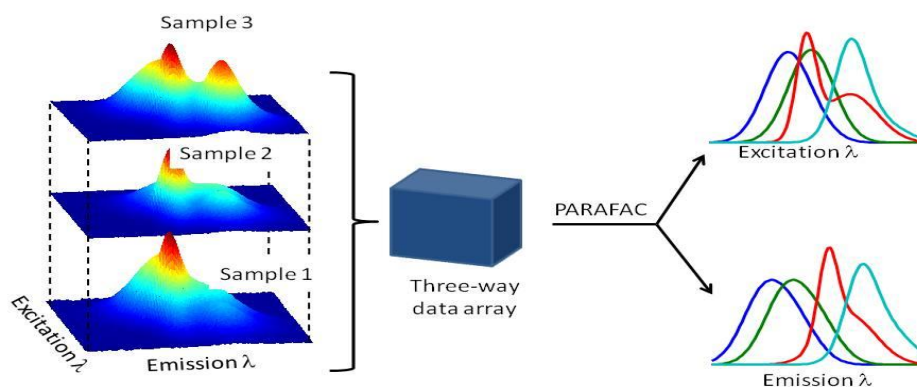


Fig. 2. Schematic representation of the PARAFAC operation, which builds a three-way data array with the different data matrices, and then decomposes the array into loadings in both instrumental data modes (in this case excitation and emission fluorescence loadings as a function of wavelength, contained in **B** and **C** matrices). Sample scores are also produced, containing information relative to constituent concentrations.

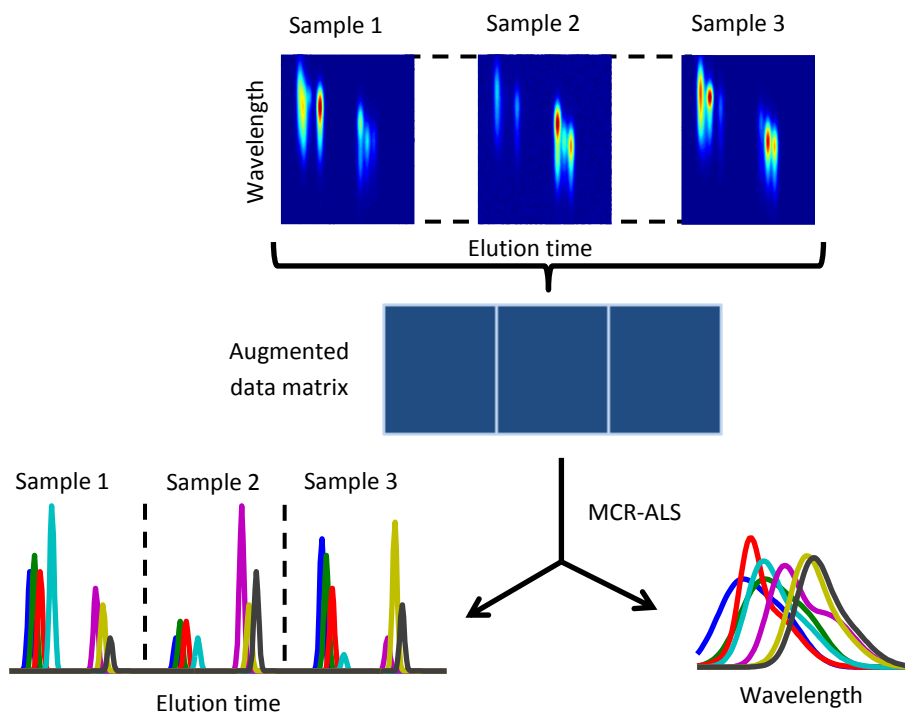


Fig. 3. Schematic representation of the operation of MCR-ALS. After building the augmented data matrix, the latter is decomposed into profiles in the augmented mode (the elution time mode in chromatographic-spectral data processing) and the spectra which are common to all samples.

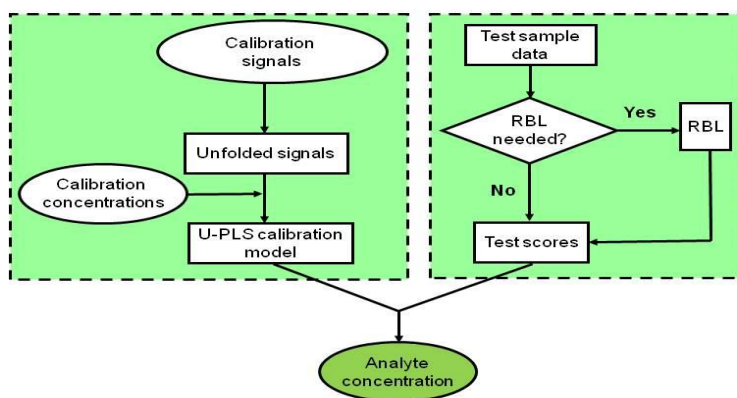


Fig. 4. Schematic representation of the U-PLS/RBL procedure.

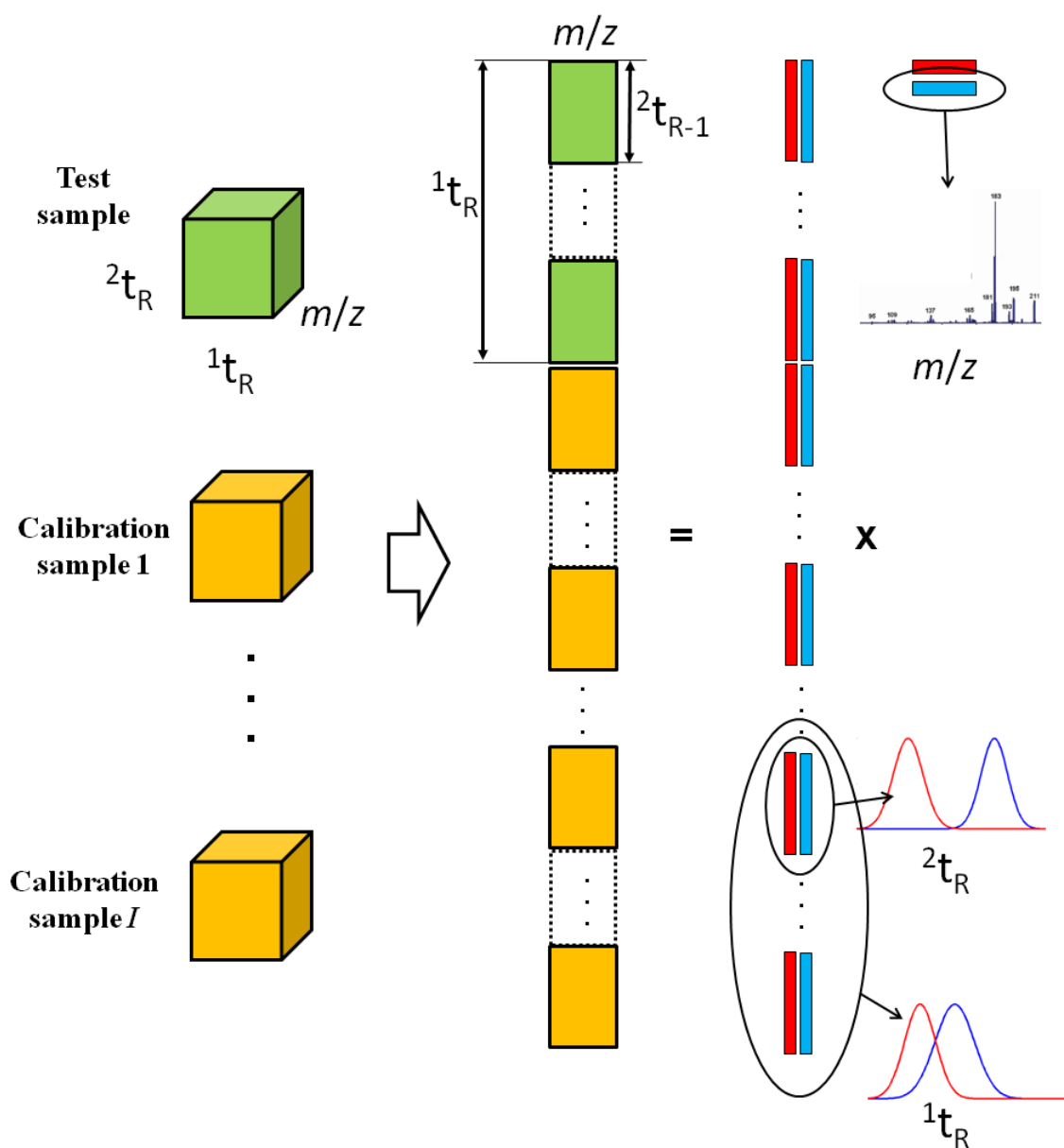


Fig. 5. Schematic representation of the MCR-ALS processing of GC-GC-MS third-order data in a two component system: each of the data arrays is first unfolded into a matrix, by concatenating the two temporal modes. The matrices are then joined into a single super-augmented matrix and processed, leading to spectra and unfolded elution time profiles. The latter can be refolded to get the individual elution time profiles along both temporal data modes. The second temporal mode is obtained directly from the retrieved temporal profiles, while the first temporal profile is obtained through the areas computed on the second-mode profile.

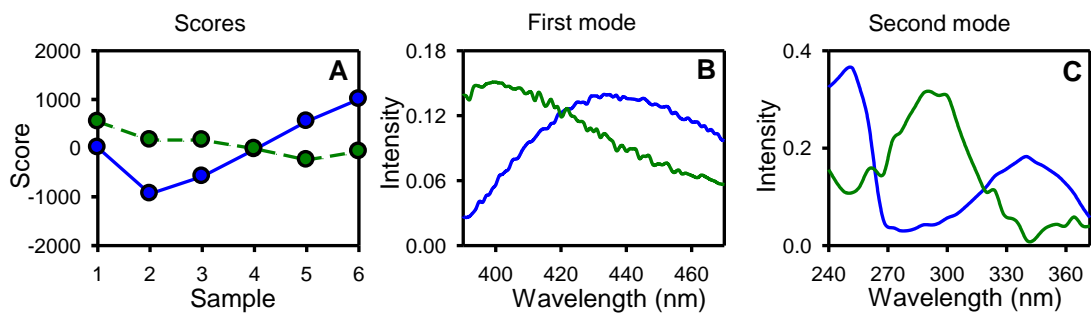


Fig. 6. PARAFAC scores (A) and loadings (B and C) for a natural stream sample added with bentazone. Blue and green lines and symbols correspond to constituents 1 and 2, respectively.

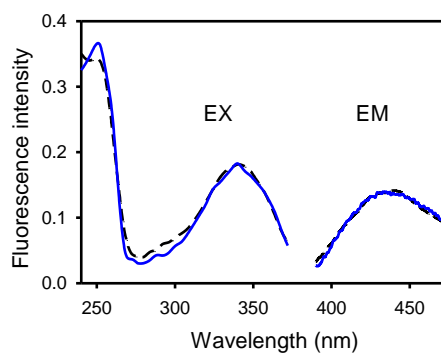


Fig. 7. Experimental excitation (EX) and emission (EM) spectra of bentazone (black dashed lines) and the corresponding PARAFAC loadings (blue solid lines). Loadings and spectra have been normalized to unit amplitude.

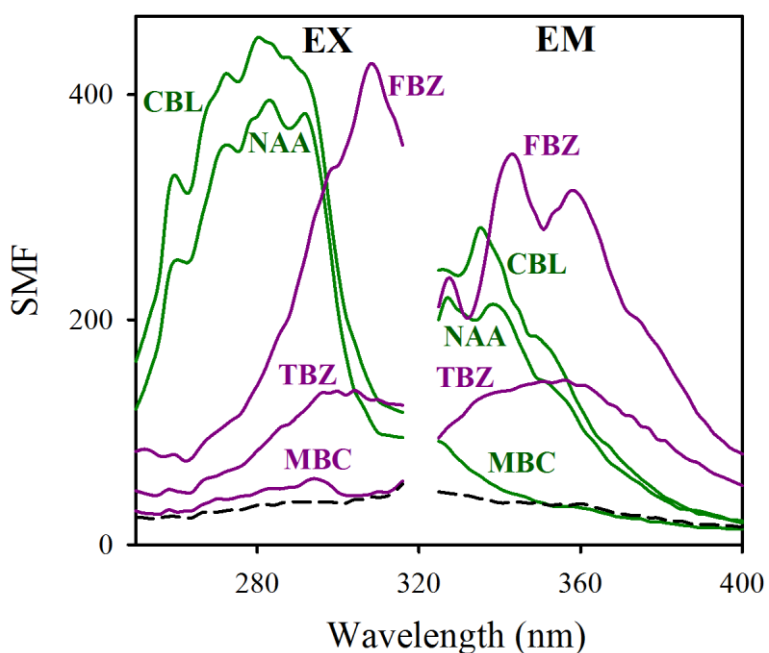


Fig. 8. Solid matrix fluorescence (SMF) excitation and emission spectra for thiabendazole (TBZ), fuberidazole (FBZ), carbaryl (CBL), 1-naphthalene acetic acid (NAA) and carbendazim (MBC) immobilized onto silica gel C18. The dashed black lines correspond to the background signals.

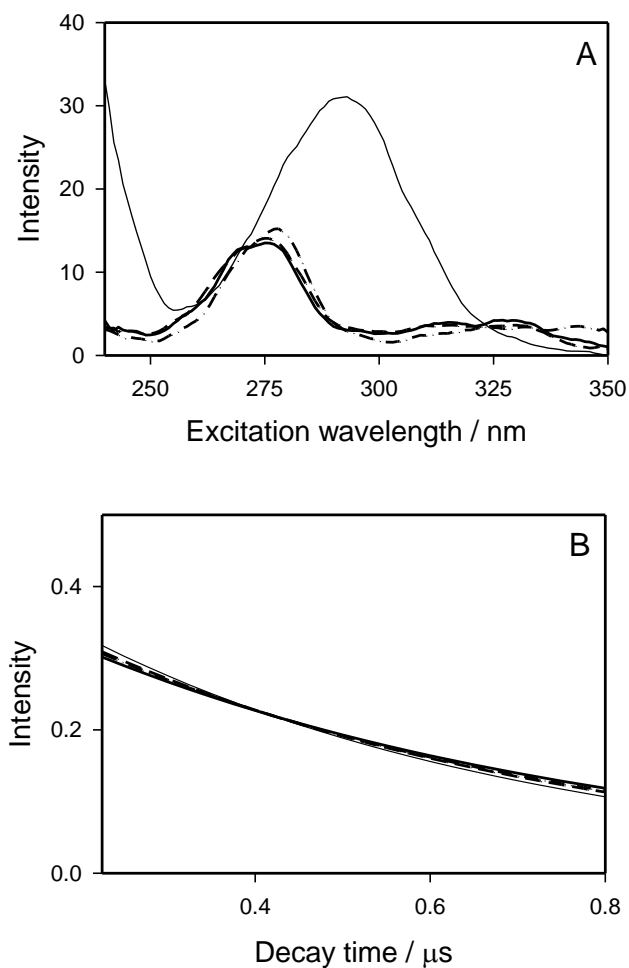


Fig. 9. A) Thick lines: excitation lanthanide-sensitized luminescence spectra for the terbium (III) complexes of the studied analytes in serum: ciprofloxacin, solid line, norfloxacin, dashed line, danofloxacin, dashed-dotted line. Thin line: the corresponding excitation spectrum for salicylate in serum. B) Time decay curves for all the constituents shown in part A), after normalization to unit length. Lines are denoted as in plot A). Reprinted with permission from [62]. Copyright 2009 Elsevier.

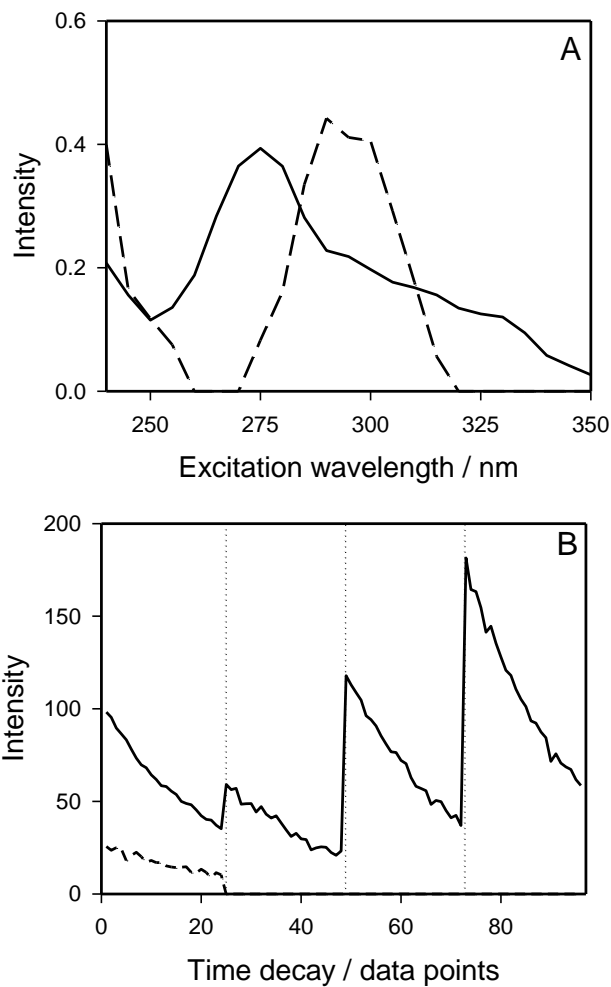


Fig. 10. MCR-ALS results for a serum sample spiked with ciprofloxacin and salicylate. A) Excitation lanthanide-sensitized luminescence profiles for the terbium (III) complexes of both sample constituents. B) Time-decay profiles for successive matrix samples in the standard addition mode. The dotted lines separate the different samples: from left to right, test sample and the results of subtracting the test sample data from the three standard additions. In all cases, the solid line indicates ciprofoxacin and the dashed line salicylate. The vertical scales are arbitrary. Reprinted with permission from [62]. Copyright 2009 Elsevier.

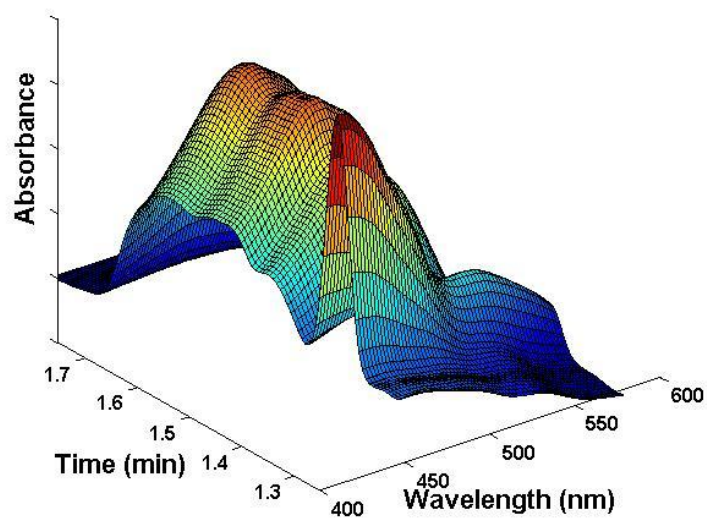


Fig. 11. Second-order data matrix generated by HPLC-DAD

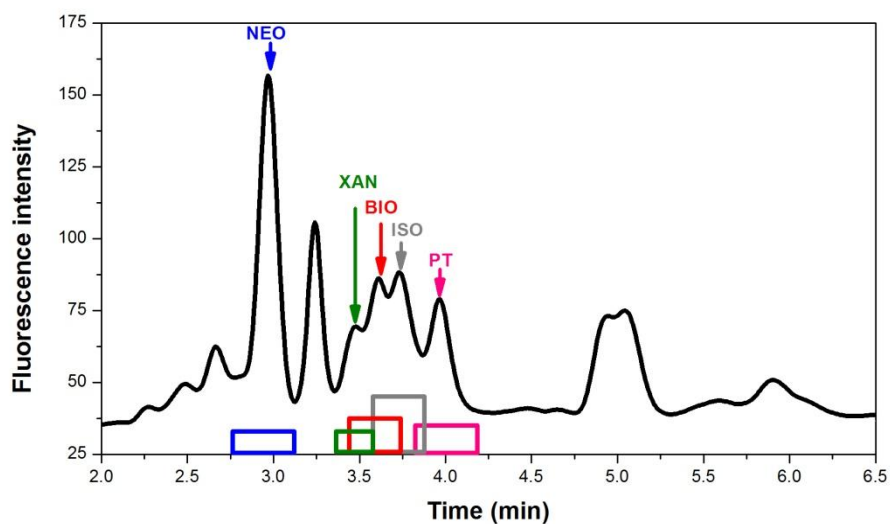


Fig. 12. Chromatogram registered at $\lambda_{exc} = 272$ nm, $\lambda_{em} = 445$ nm and elution time between 2.0 and 6.5 min for a urine sample after spiking with the five analytes. The boxes correspond to the five regions used the MCR-ALS analysis.

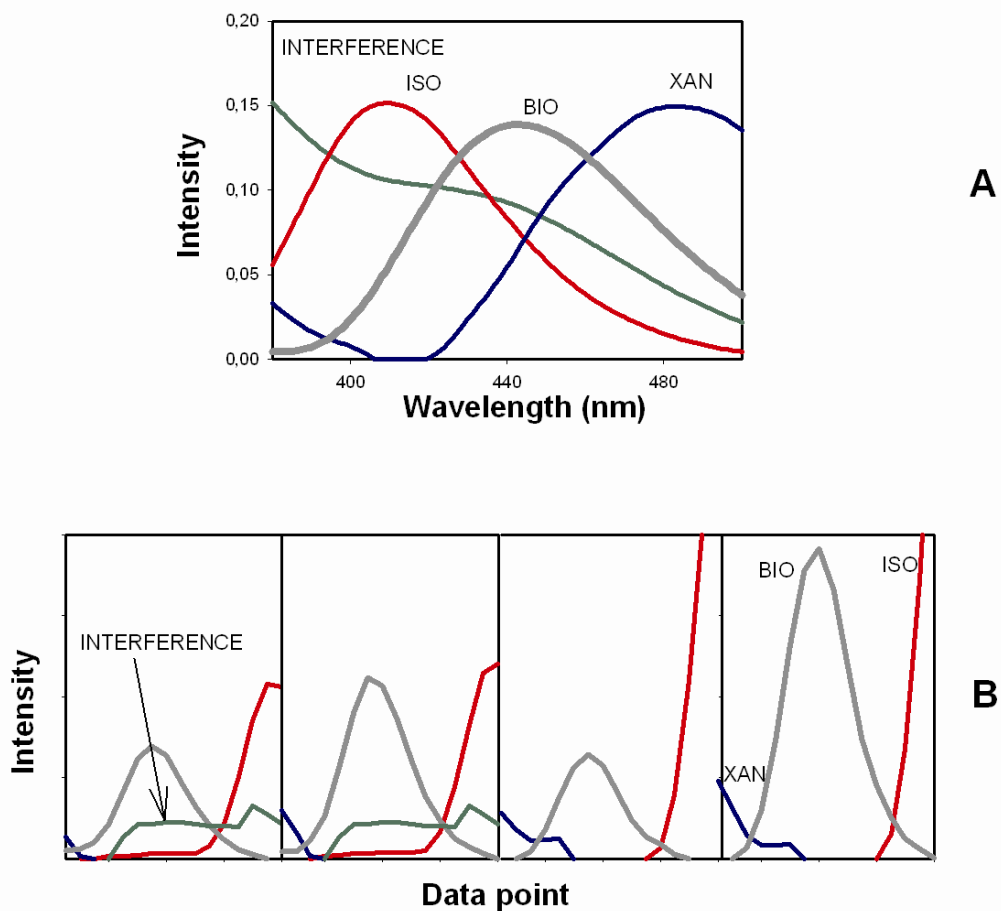


Fig. 13. A) MCR-ALS retrieved spectral profiles in region number 3: BIO, ISO, XAN and one interference, as indicated. B) Successive elution time profiles, corresponding to the MCR-ALS analysis of 4 samples in region 3, which includes the BIO peak in two urine samples from a healthy adult (two spiked concentrations), and two pure standard solutions (3.0 and 8.0 ng mL⁻¹, respectively). The remaining three profiles correspond to ISO, XAN, and one interferent.

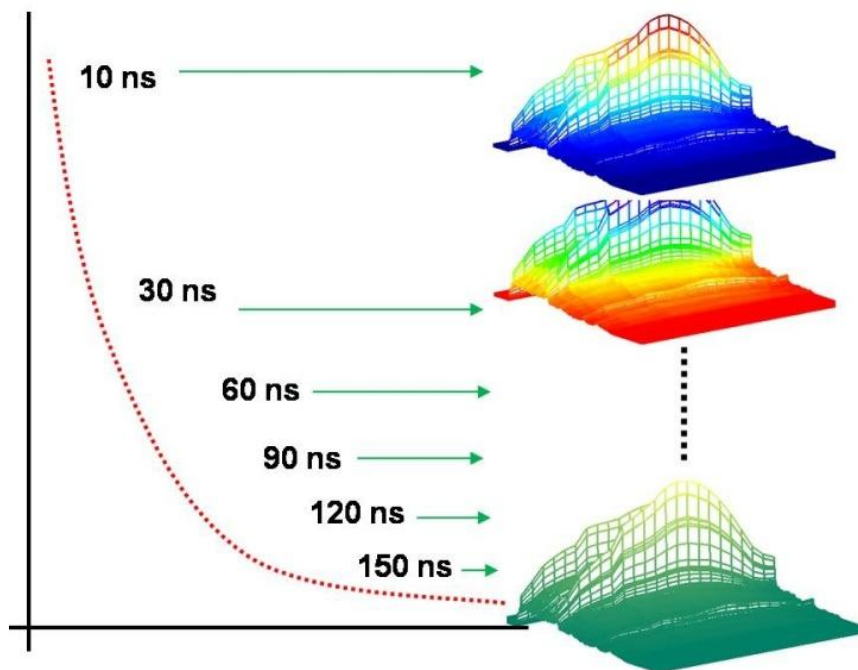


Fig. 14. Schematic representation of laser-excited time-resolved EEM fluorescence data collection.

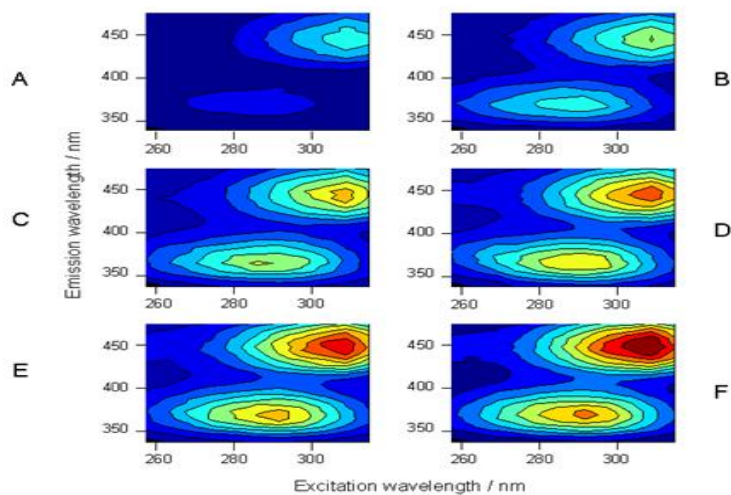


Fig. 15. Contour plots of the EEMs for an aqueous solution containing methotrexate and leucovorin as a function of the time of permanganate oxidation. The times selected for illustrating the kinetic evolution of the EEMs are (in min): A) 0, B) 2.4, C) 4.8, D) 7.2, E) 9.6 and F) 10.8. Fluorescence intensity has been coded in colors, with deep blue indicating the lowest value, and deep red the largest one. Reprinted with permission from [75]. Copyright 2004 American Chemical Society.

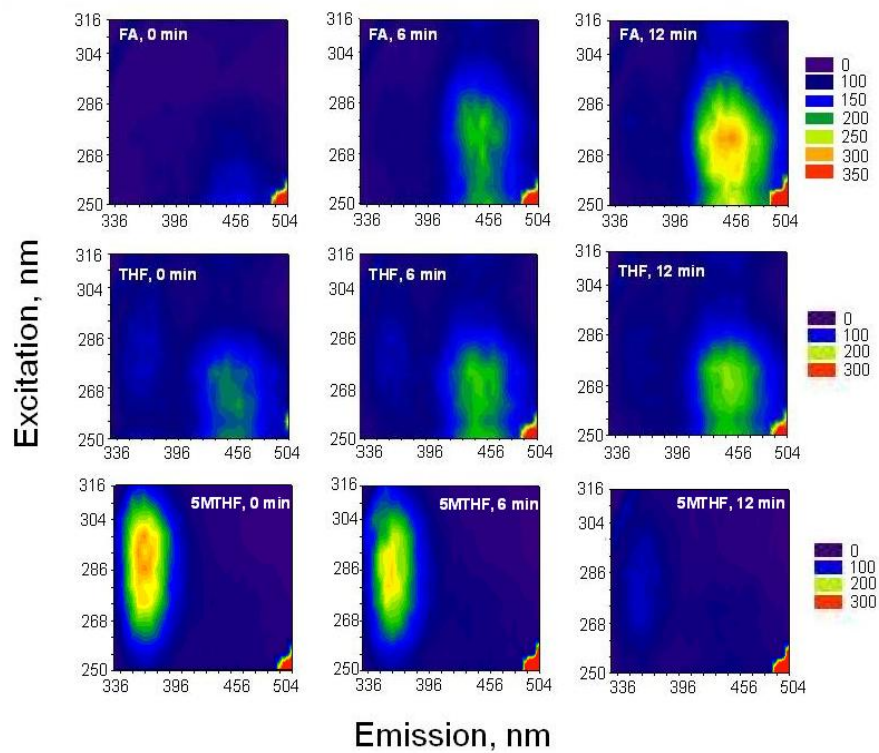


Fig. 16. Evolution of the contour plot of the excitation-emission matrices with the irradiation time, of solutions of folic acid (FA), tetrahydrofolic acid (THF) and 5-methyltetrahydrofolic acid (5-MTF). Reprinted with permission from [77]. Copyright 2008 Elsevier.

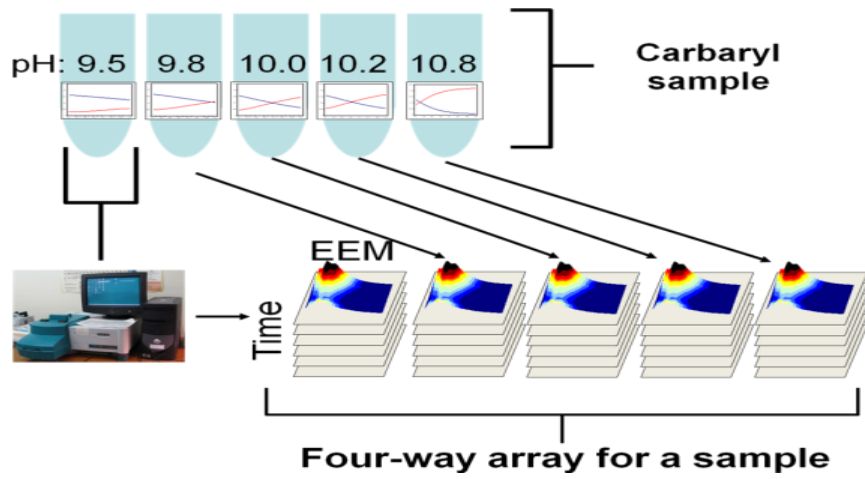


Fig. 17. Building of a four-way data array for a single sample of carbaryl.

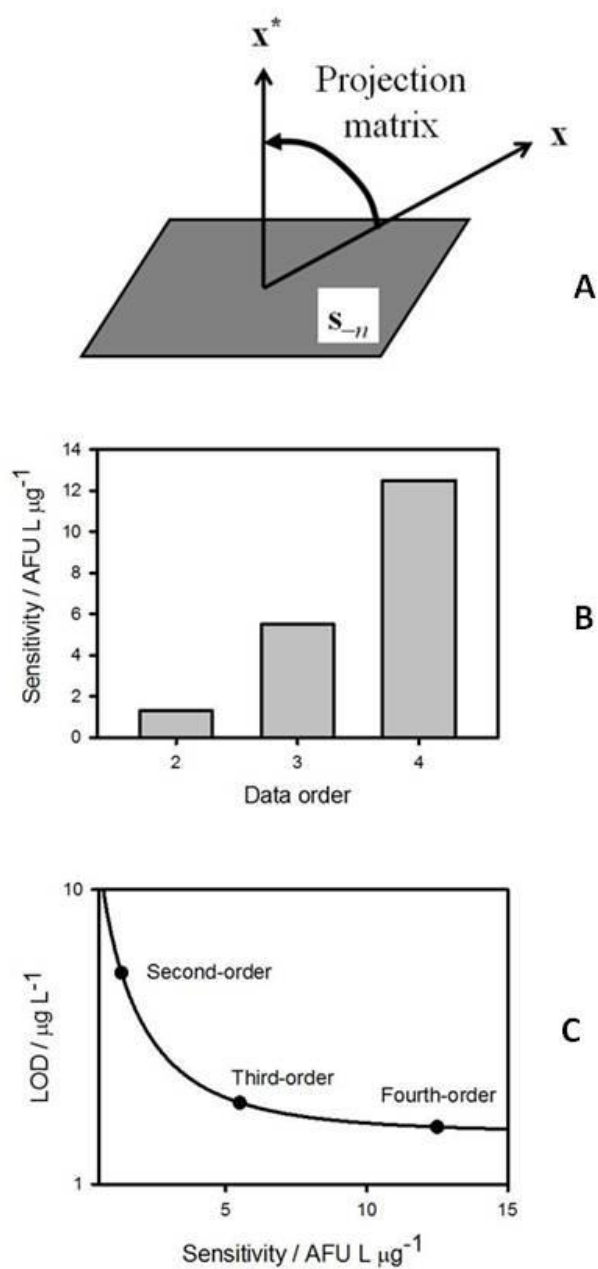


Fig. 18. A) Schematic illustration of the projection of a signal (\mathbf{x}) orthogonal to the space spanned by the interferences, leading to the net analyte signal (\mathbf{x}^*). B) Sensitivity as a function of order in an experimental system (see text). C) LOD as a function of sensitivity for various orders (see text).